FAST MULTIPLE-PRECISION EVALUATION OF ELEMENTARY FUNCTIONS

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Abstract

Let $f(x)$ be one of the usual elementary functions (exp, log, artan, sin, cosh, etc.), and let $M(n)$ be the number of single-precision operations required to multiply n-bit integers. It is shown that $f(x)$ can be evaluated, with relative error $O(2^{-n})$, in $O(M(n)\log(n))$ operations as $n \to \infty$, for any floating-point number x (with an n-bit fraction) in a suitable finite interval. From the Schönhage-Strassen bound on $M(n)$, it follows that an n-bit approximation to $f(x)$ may be evaluated in $O(n\log^2(n)\log\log(n))$ operations. Special cases include the evaluation of constants such as π, e, and e^π. The algorithms depend on the theory of elliptic integrals, using the arithmetic-geometric mean iteration and ascending Landen transformations.

Comments

Only the Abstract is given here. The full paper appeared as [5]. For related work see [1, 2, 4].

References

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