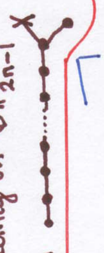
17|10|12

Flag manifolds G/Q with transitively acting real form G_0 are:-
- G_0 compact
- $G_0 = SL(n,\mathbb{H})$ acting on $\mathbb{CP}_{2n-1}$
- $G_0 = SO(2n-1,1)$ acting on



[From last time

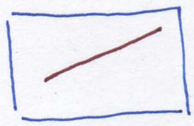
$SO(3,1) \simeq SL(2,\mathbb{C})$ but also as computational tool!

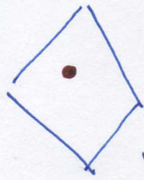
Using twistor to produce self-dual compact Riemannian manifolds
- Poon 1986 $\mathbb{CP}_2 \# \mathbb{CP}_2$
- Donaldson & Friedman 1989 $X_1 \# X_2$
- Floer 1991
- LeBrun 1991 $\underbrace{\mathbb{CP}_2 \# \cdots \# \mathbb{CP}_2}_{n}$ explicit!
- LeBrun & Singer 1993 scalar flat Kähler
- " " 1994 Kummer K3 surface (CY)
- LeBrun .... 2004..... $\overbrace{\qquad}^{\text{optimal metrics}}$

What is the Penrose transform?

Recall

$\mathbb{F}$, $\mathbb{CP}_3$, $\mathbb{M} = Gr_2(\mathbb{C}^4)$





$\mathbb{CP}_3 = SL(4,\mathbb{C}) \Big/ \left\{\begin{pmatrix} * & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{pmatrix}\right\} = \overset{}{\times}\!\!-\!\bullet\!-\!\bullet$

$\mathbb{M} = SL(4,\mathbb{C}) \Big/ \left\{\begin{pmatrix} * & * & * & * \\ * & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & * & * \end{pmatrix}\right\} = \bullet\!-\!\times\!-\!\bullet$

EPW: $0 \to P \to \mathbb{C}^4 \to \mathbb{C}^4/P \to 0$ on $\mathbb{M}$

$\rightsquigarrow 0 \to (\mathbb{S}')^* \to \mathbb{C}^4 \to \mathbb{S} \to 0$

$0 \to \mathcal{O}_{A'} \to \mathcal{O}^{\alpha} \to \mathcal{O}^{A} \to 0$

Tangent bundle $\operatorname{Hom}(P, \mathbb{C}^4/P) \rightsquigarrow$
$\mathbb{C}^4/P \otimes P^* = \mathbb{S} \otimes \mathbb{S}'$ spinors

$$\underbrace{Z \xleftarrow{\mu} F \xrightarrow{\nu} \mathbb{M} \overset{\text{open}}{\supset} U}_{\text{holomorphic}}$$

$H^r(\underset{\tilde{U}}{\underset{\|}{\mu(\nu^{-1}(U))}}, \mathcal{O}(E)) \Rightarrow$ data on U

E: vector bundle on Z

$0 \to \mu^{-1}\mathcal{O} \to \overset{\mathcal{O}}{\overset{\|}{\Omega^0_\mu}} \xrightarrow{d_\mu} \Omega^1_\mu \xrightarrow{d_\mu} \Omega^2_\mu \to \cdots \rightsquigarrow E_1^{p,q} = \Gamma(U, \nu_*^q(\Omega^p_\mu(E))) \Rightarrow H^{p+q}(\tilde{U}, \mathcal{O}(E))$

(under some conditions)

EG classical case $U = \mathbb{M}^{++} \Rightarrow \mu(\nu^{-1}(U)) = \mathbb{P}^+$ $\quad \Omega^1_\mu = \overset{1\ -2\ 1}{\times\!-\!\times\!-\!\bullet}$ $\quad \Omega^2_\mu = \overset{2\ -3\ 0}{\times\!-\!\times\!-\!\bullet}$

$\overset{0\ 0\ 0}{\bullet\!-\!\times\!-\!\bullet} \to \overset{1\ -2\ 1}{\bullet\!-\!\times\!-\!\bullet} \to \overset{2\ -3\ 0}{\bullet\!-\!\times\!-\!\bullet} \to p \quad \Rightarrow H^1(\mathbb{P}^+, \mathcal{O}) \simeq \dfrac{\ker d_+ : \Lambda^1 \to \Lambda^2_+}{\operatorname{im} d : \Lambda^0 \to \Lambda^1}$ on $\mathbb{M}^{++}$ pot/gauge

$\quad \| \qquad\qquad \| \qquad\qquad \|$
$\quad \Lambda^0 \qquad\quad \Lambda^1 \qquad\quad \Lambda^2_+$

$\uparrow q$

$\simeq$ positive frequency a.s.d. Maxwell

[M = Shilov boundary of $\mathbb{M}^{++}$ = tube over future cone]

$E = \overset{-4\ 0\ 0}{\times\!-\!\bullet\!-\!\bullet} \rightsquigarrow \overset{2\ -3\ 0}{\bullet\!-\!\times\!-\!\bullet} \quad \overset{1\ -4\ 1}{\bullet\!-\!\times\!-\!\bullet} \quad \overset{0\ -4\ 0}{\bullet\!-\!\times\!-\!\bullet}$

$\qquad\qquad\qquad \Lambda^2_+ \qquad \Lambda^3 \qquad \Lambda^4$

$0 \longrightarrow 0 \longrightarrow 0 \longrightarrow p$

$\boxed{\nu_*^1 \overset{-3\ -2\ 1}{\times\!-\!\times\!-\!\bullet} = \overset{2\ -3\ 1}{\bullet\!-\!\times\!-\!\bullet}}$

$\Rightarrow H^1(\mathbb{P}^+, \mathcal{O}(-4)) \simeq$ +ve freq s.d. Maxwell

Generally $H^1(\mathbb{P}^+, \mathcal{O}(-k-2)) \simeq$ helicity $k/2$ massless fields. EG $k=0$ scalar fields

Einstein bundle $\overset{0\ 1\ 0}{\times\!-\!\bullet\!-\!\bullet} \simeq \Lambda^1 \otimes \overset{2\ 0\ 0}{\times\!-\!\bullet\!-\!\bullet}$

curved setting $\rightsquigarrow$ non-linear graviton

NB
- twistor transform (not Penrose transform)
- $H^1(\tilde{U}, \mathcal{O}) \rightsquigarrow$ non-linear version Ward transform