Theorems Beginning with P

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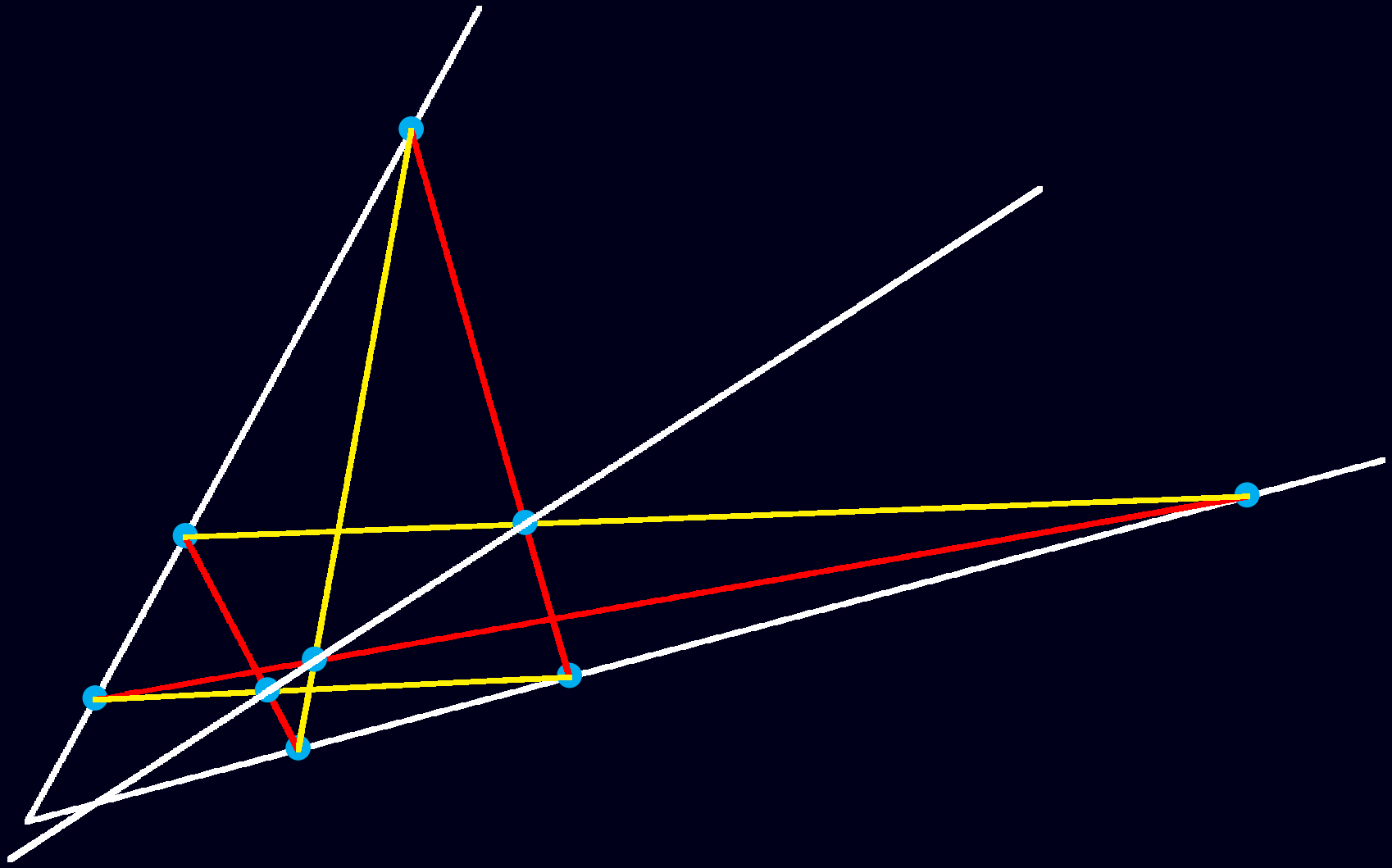
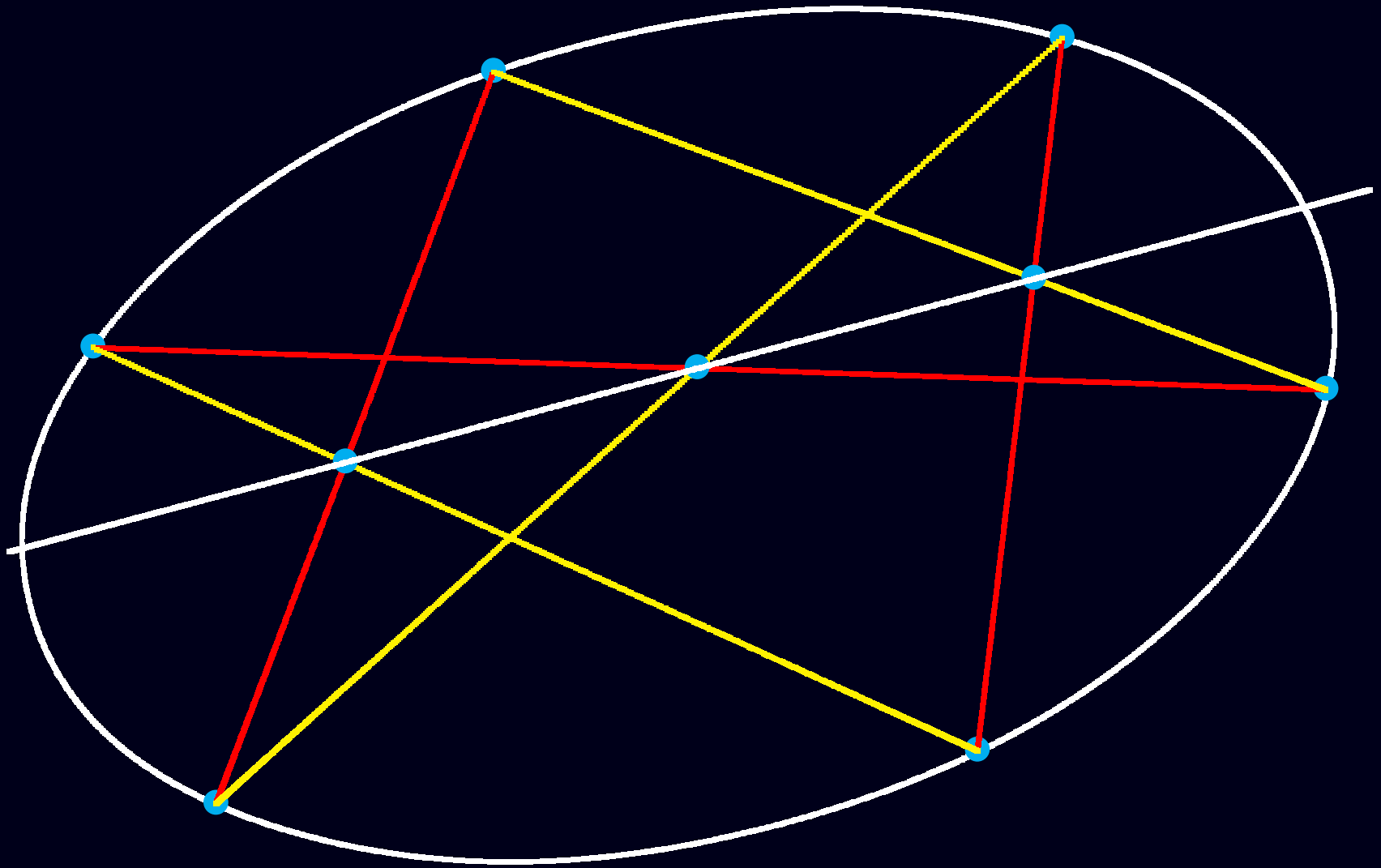
Results from Plane Geometry

- Pappus' Theorem: Pappus of Alexandria ($\sim$290–$\sim$350)
- Pascal's Theorem: Blaise Pascal (1623–1662)
- Poncelet's Theorem: Jean-Victor Poncelet (1788–1867)
- Penrose's Theorem: Roger Penrose (1931–)
- Desargues' Theorem: Girard Desargues (1591–1661)
- Steiner's Porism: Jakob Steiner (1796–1863)
- Butterfly Theorem

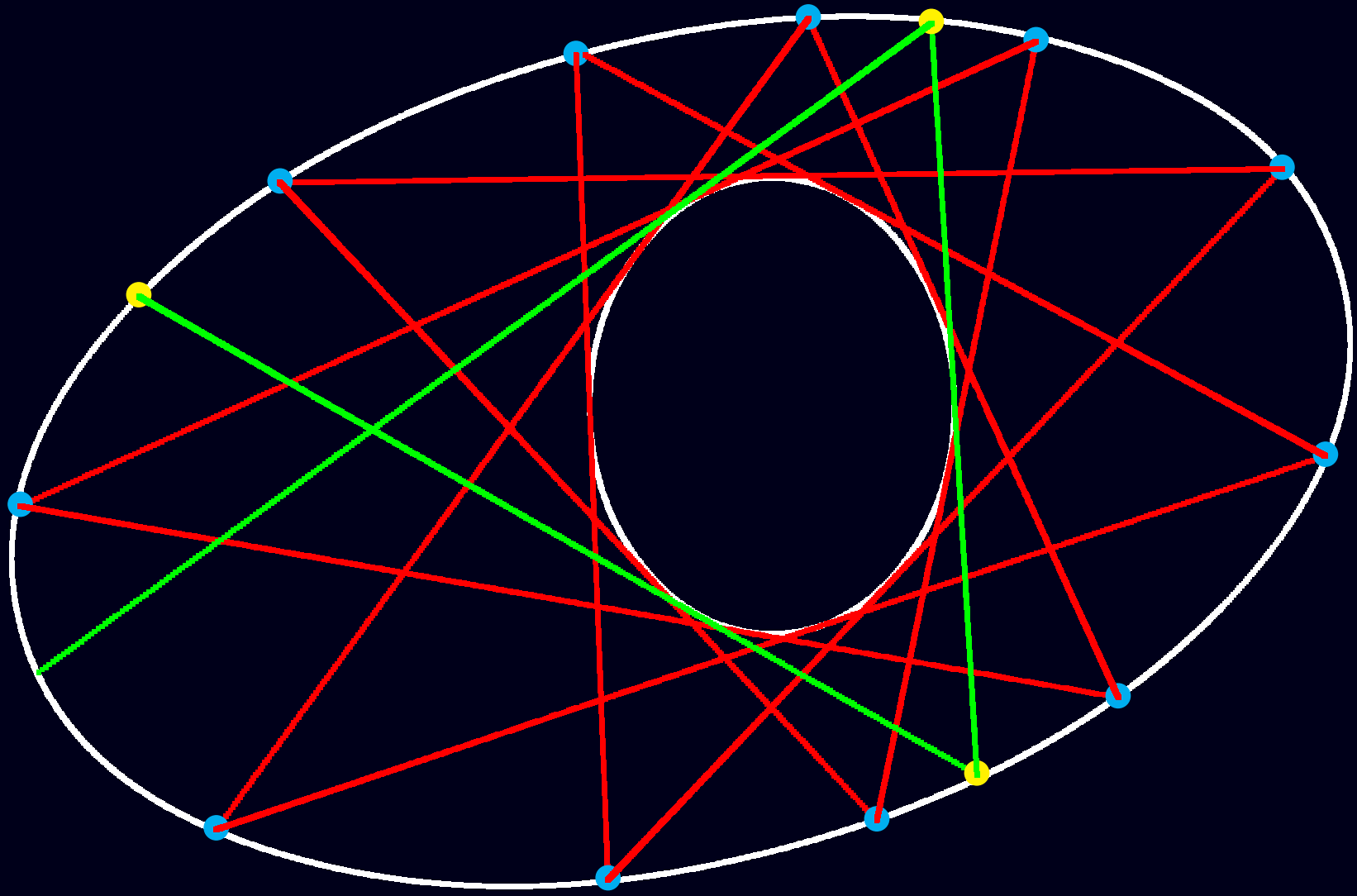
Pappus' Theorem



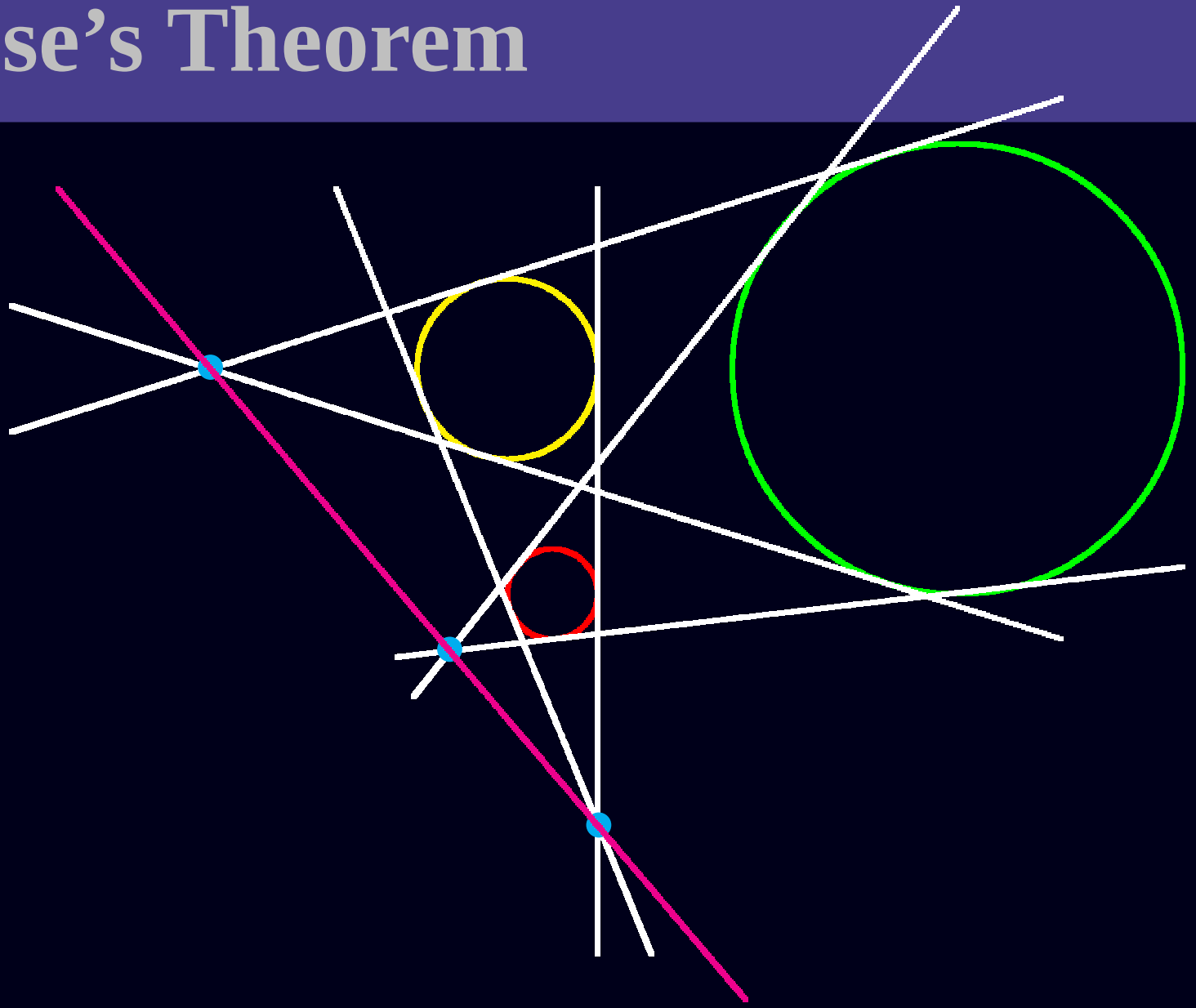
Pascal's Theorem



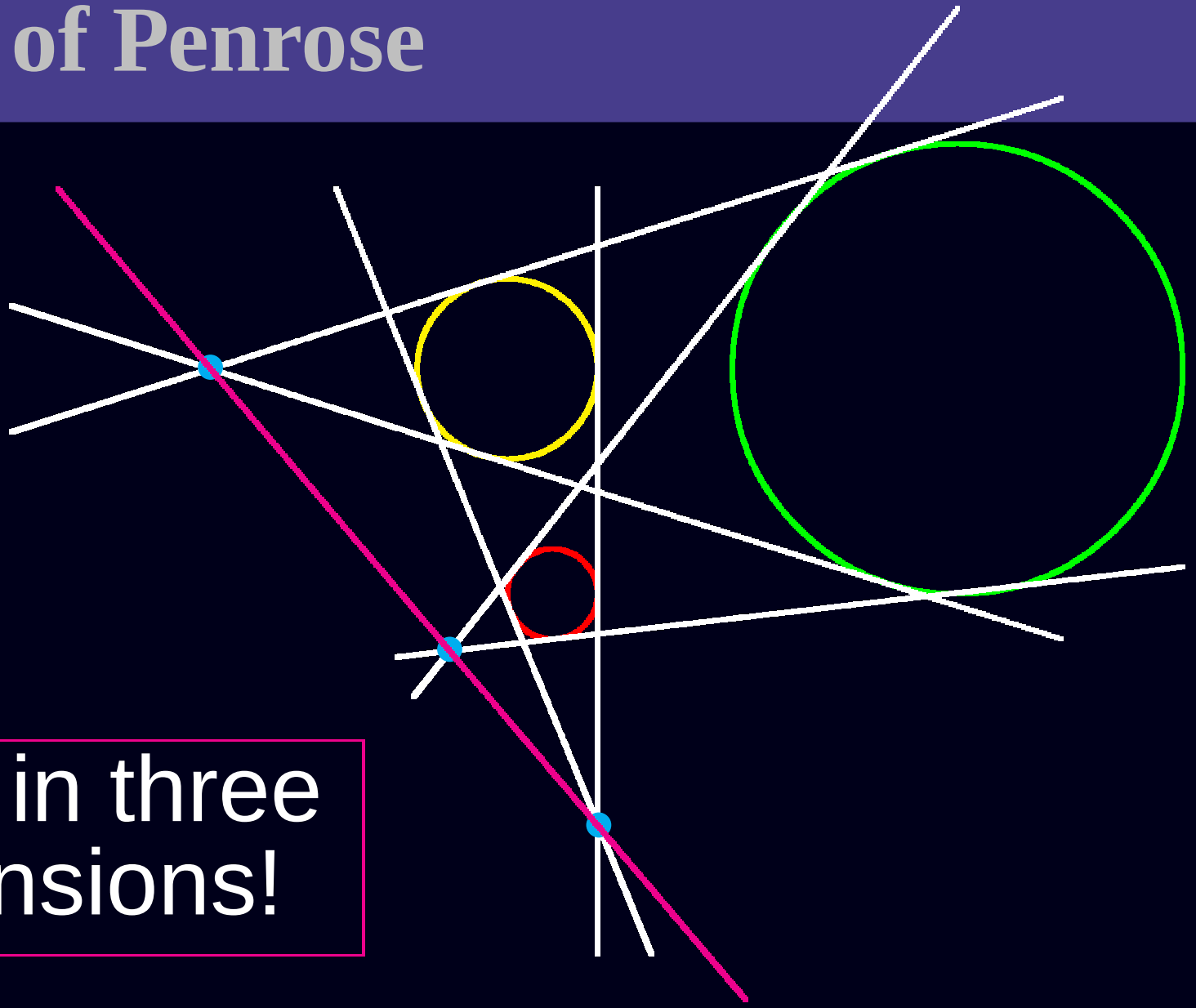
Poncelet's Theorem



Penrose's Theorem

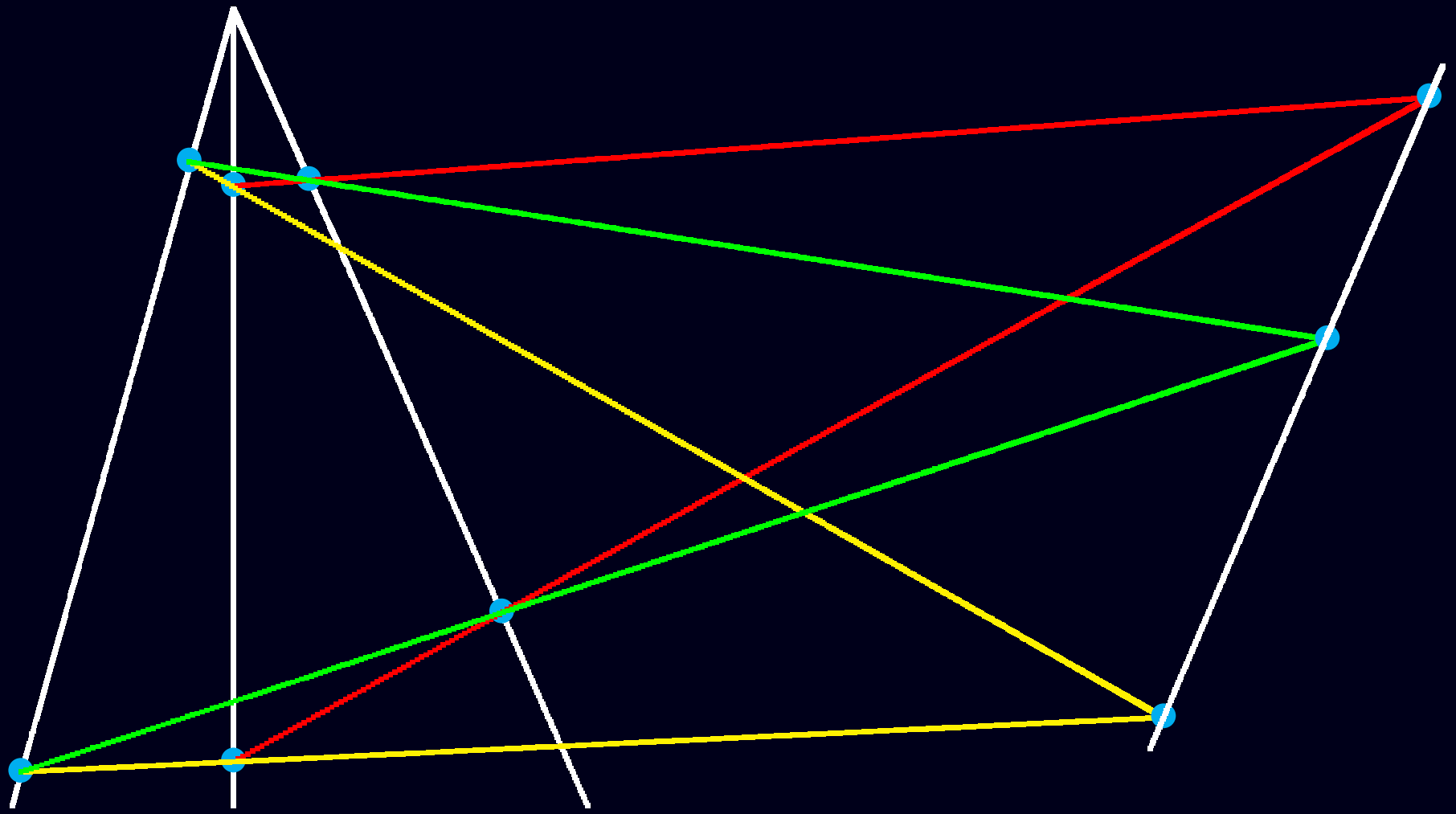


Proof of Penrose

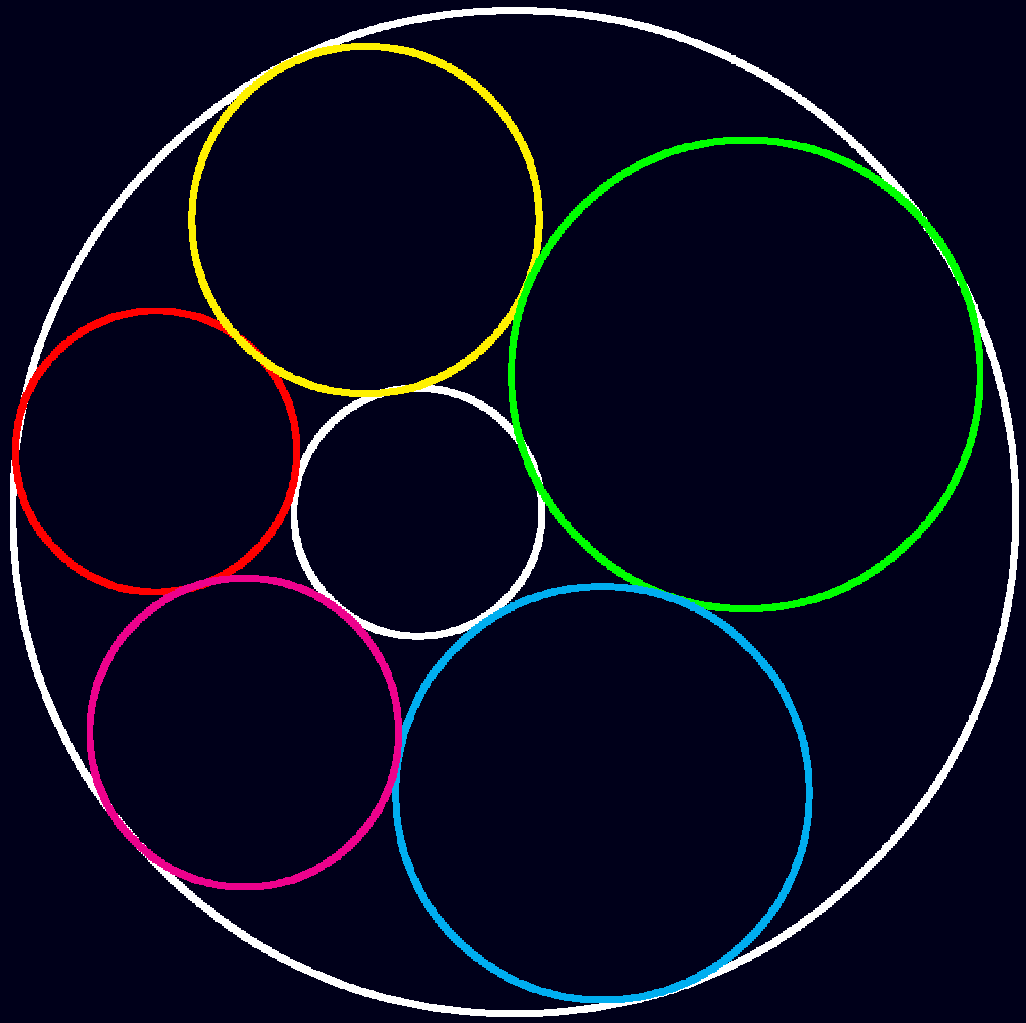


View in three dimensions!

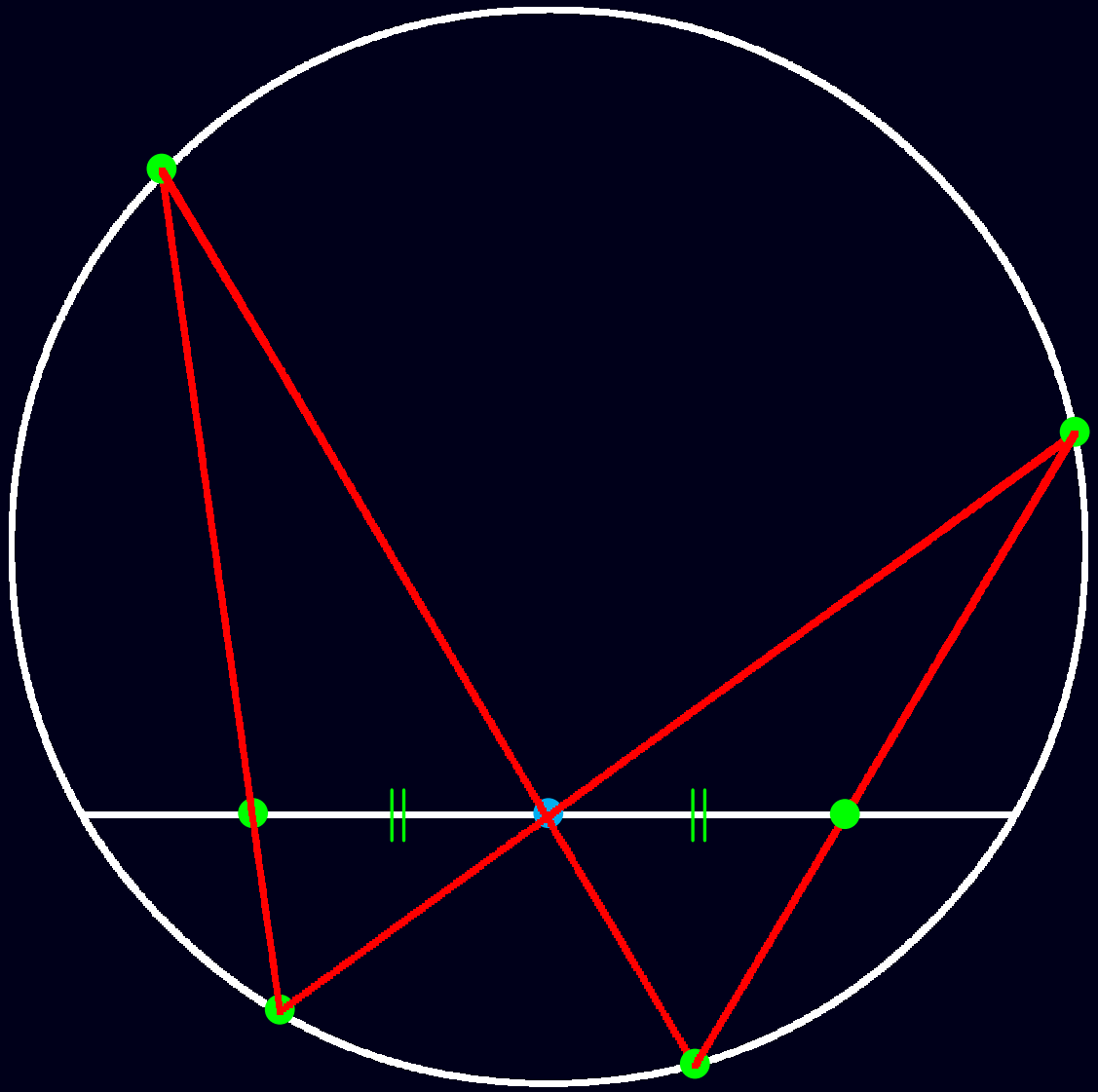
Desargues' Theorem



Steiner’s Porism

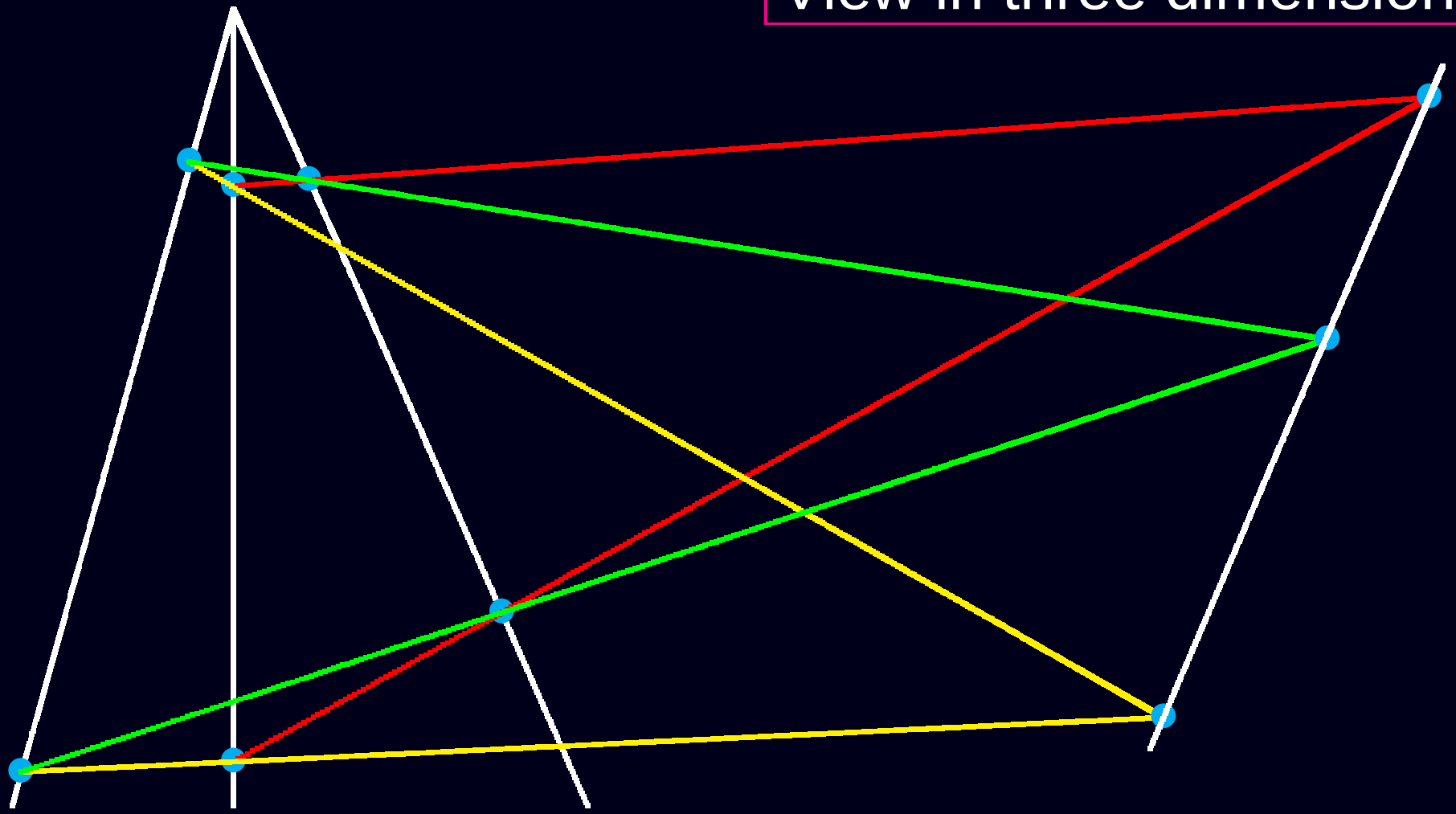


Butterfly Theorem

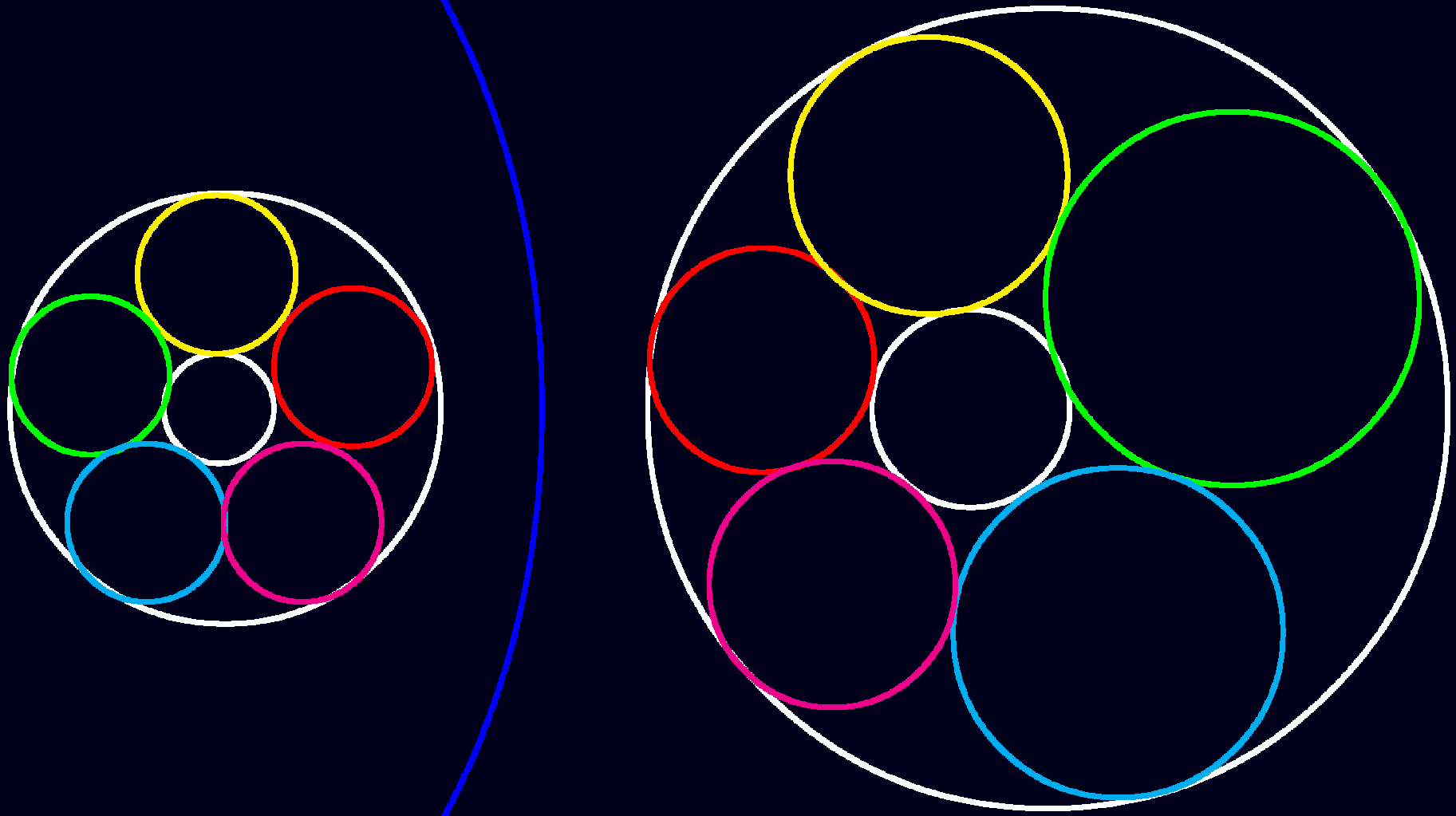


Proof of Desargues

View in three dimensions!

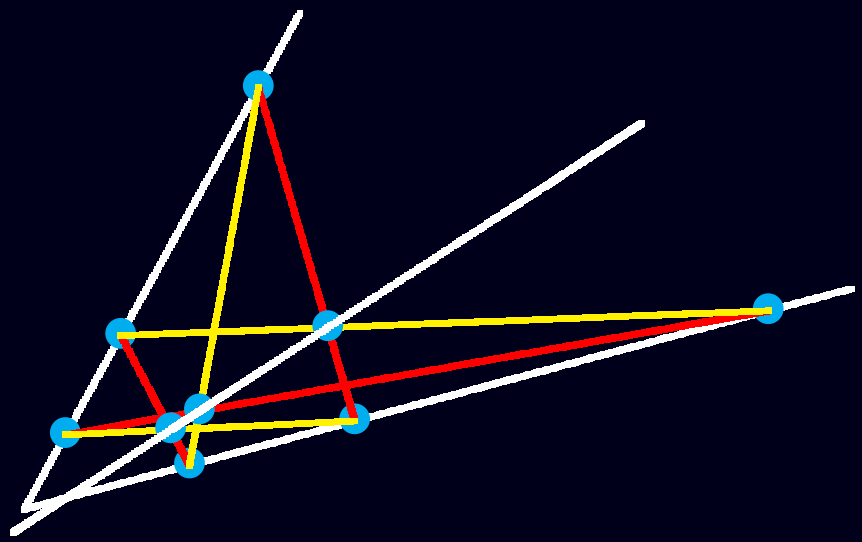


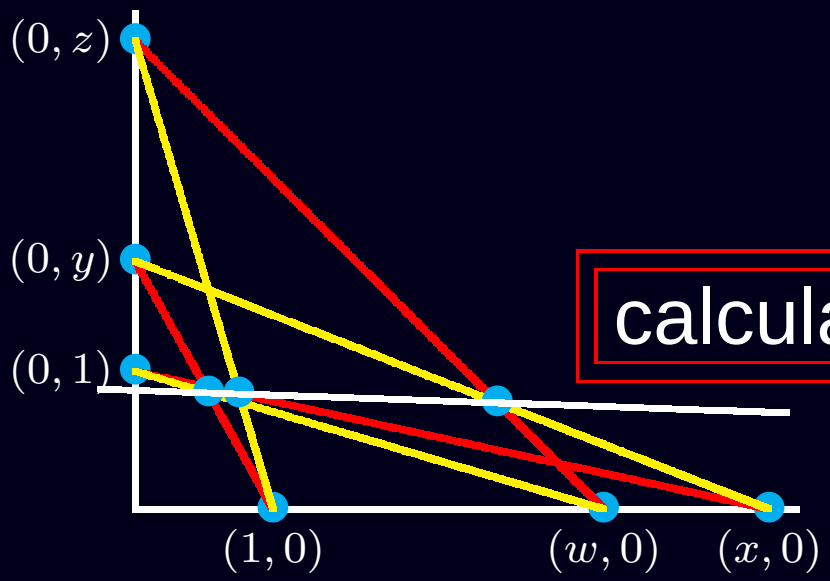
Proof of Steiner



Proof of Pappus

Brute force proof: normalise!





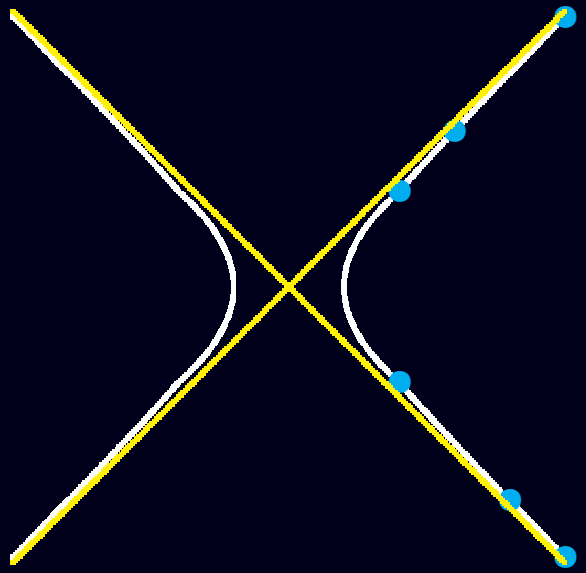
Elegant proof: deduce from Pascal!

From Pascal to Pappus

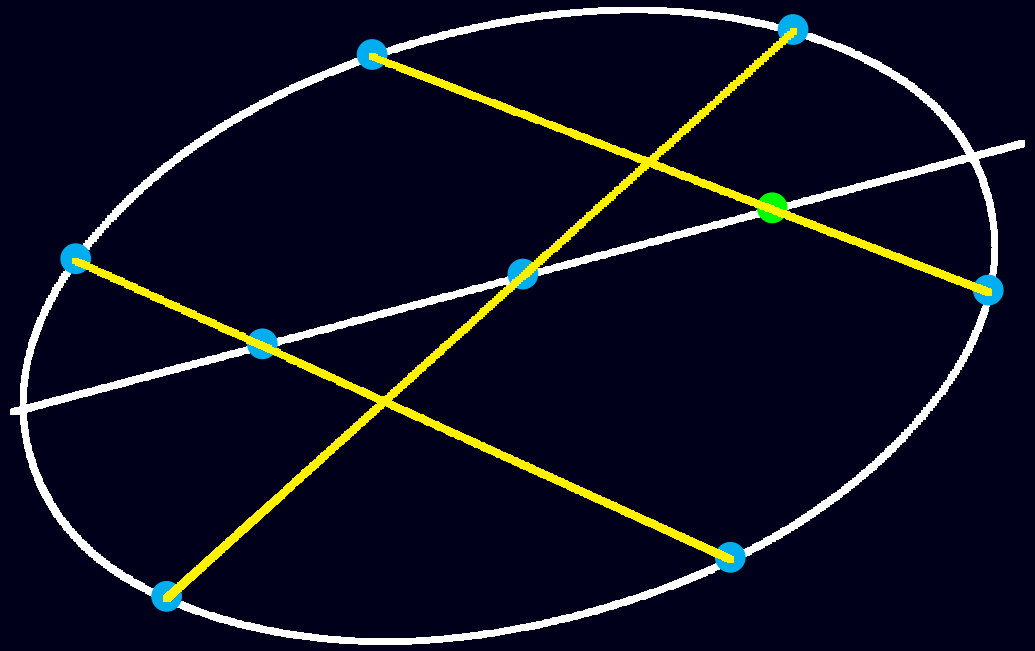
Some algebra!

- ellipse $x^2 + b^2y^2 = r^2$
- hyperbola $x^2 - b^2y^2 = r^2$
- two lines $(x - by)(x + by) = 0$

Picture

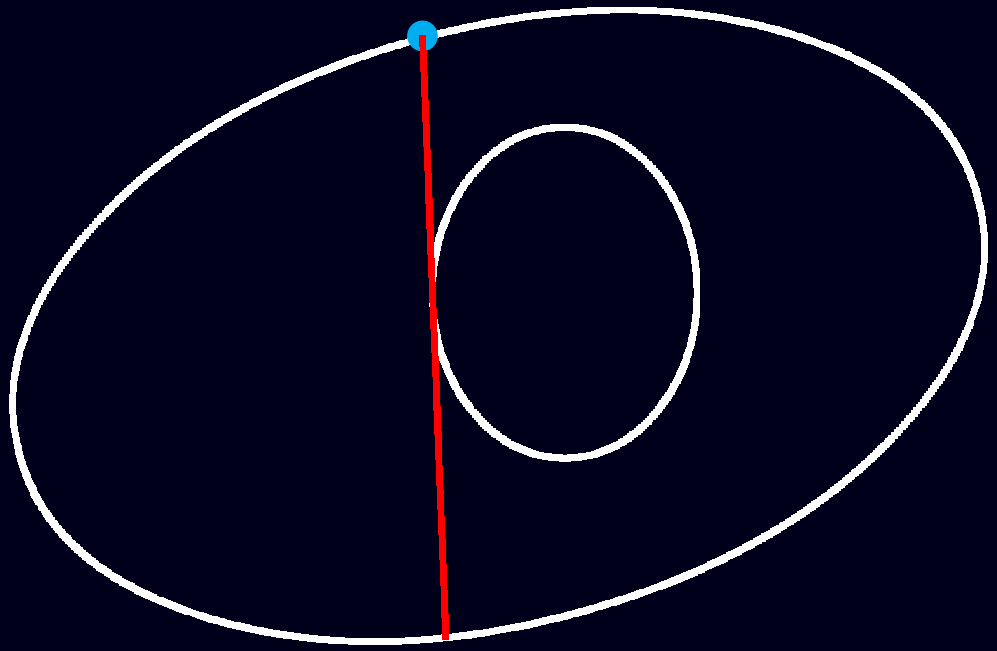


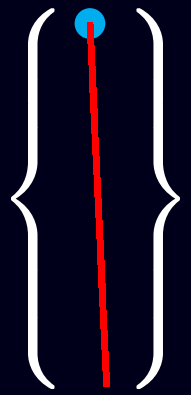
Proof of Pascal



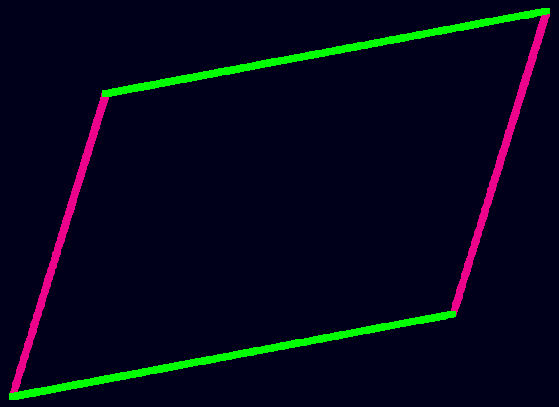
- Ellipse $\cup$ Line $\overset{\text{perturb}}{\longmapsto}$ Cubic! $\cap$ Lines
- Use complex numbers!
- Add points at infinity!
- Doughnut $\ncong$ Sphere: topology!

Proof of Poncelet





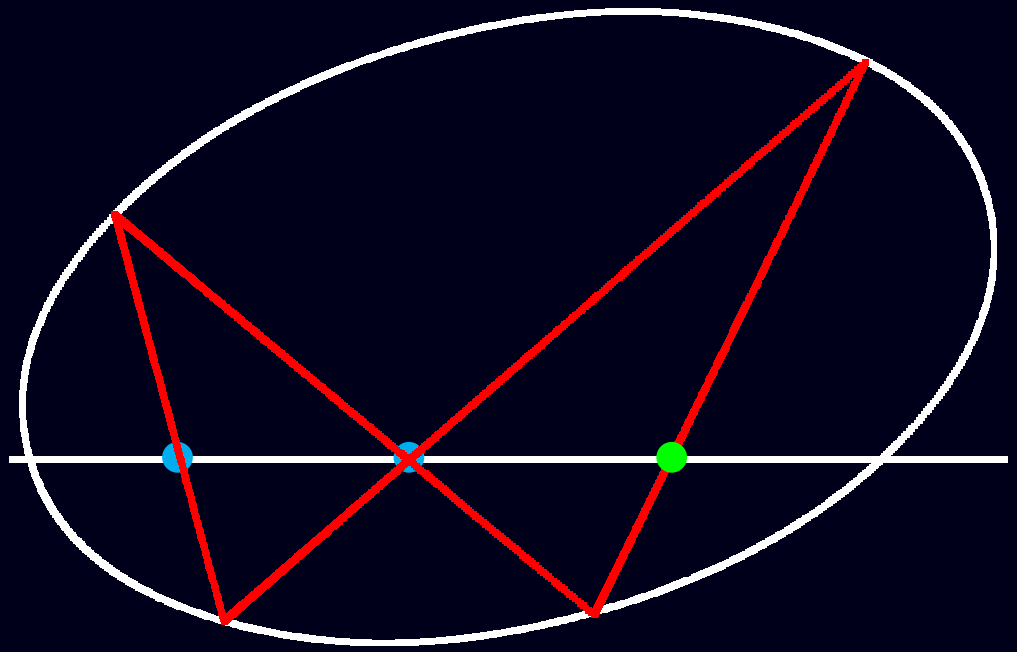
$=$ Doughnut!



One
Poncelet
Step
$\|$
Doughnut
Translation

Proof of Butterfly

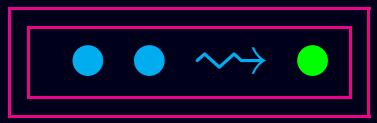
Prove a more general theorem!



Ellipse $\cup$ Line $=$ Doughnut

Doughnut is a Group!

Colinearity $\equiv$ Zero Sum!



Further Reading

- http://www.cut-the-knot.org/geometry.shtml
- Leopold Flatto, Poncelet's Theorem, American Mathematical Society 2009
- http://www.ima.umn.edu/%7Earnold/moebius/

THANK YOU

THE END