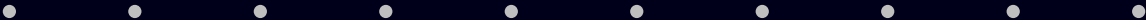




# Theorems Beginning with P

Michael Eastwood

Australian National University

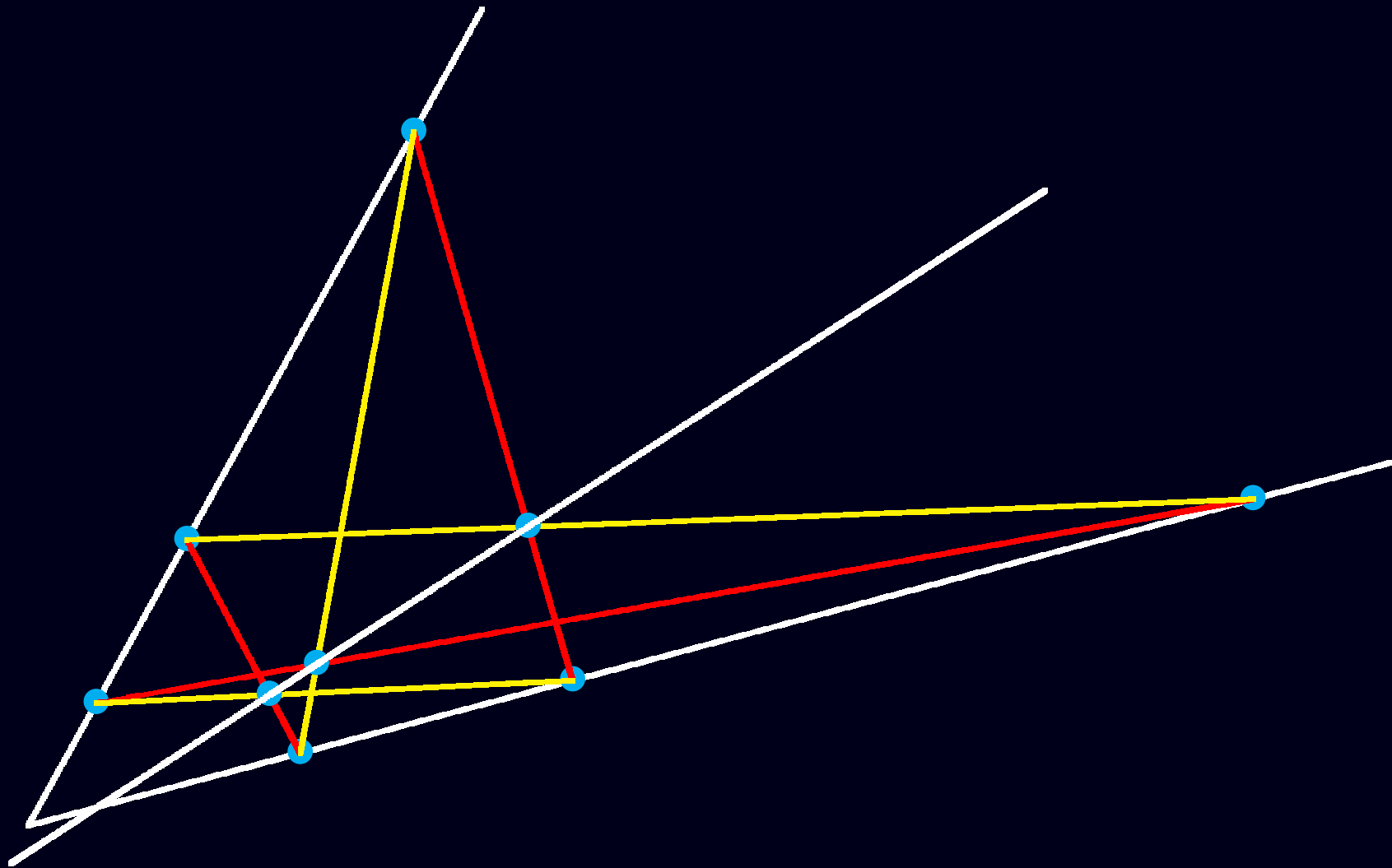


# Results from Plane Geometry

- Pappus' Theorem: Pappus of Alexandria (~290–~350)
- Pascal's Theorem: Blaise Pascal (1623–1662)
- Poncelet's Theorem: Jean-Victor Poncelet (1788–1867)
- Penrose's Theorem: Roger Penrose (1931– )
- Desargues' Theorem: Girard Desargues (1591–1661)
- Steiner's Porism: Jakob Steiner (1796–1863)
- Butterfly Theorem

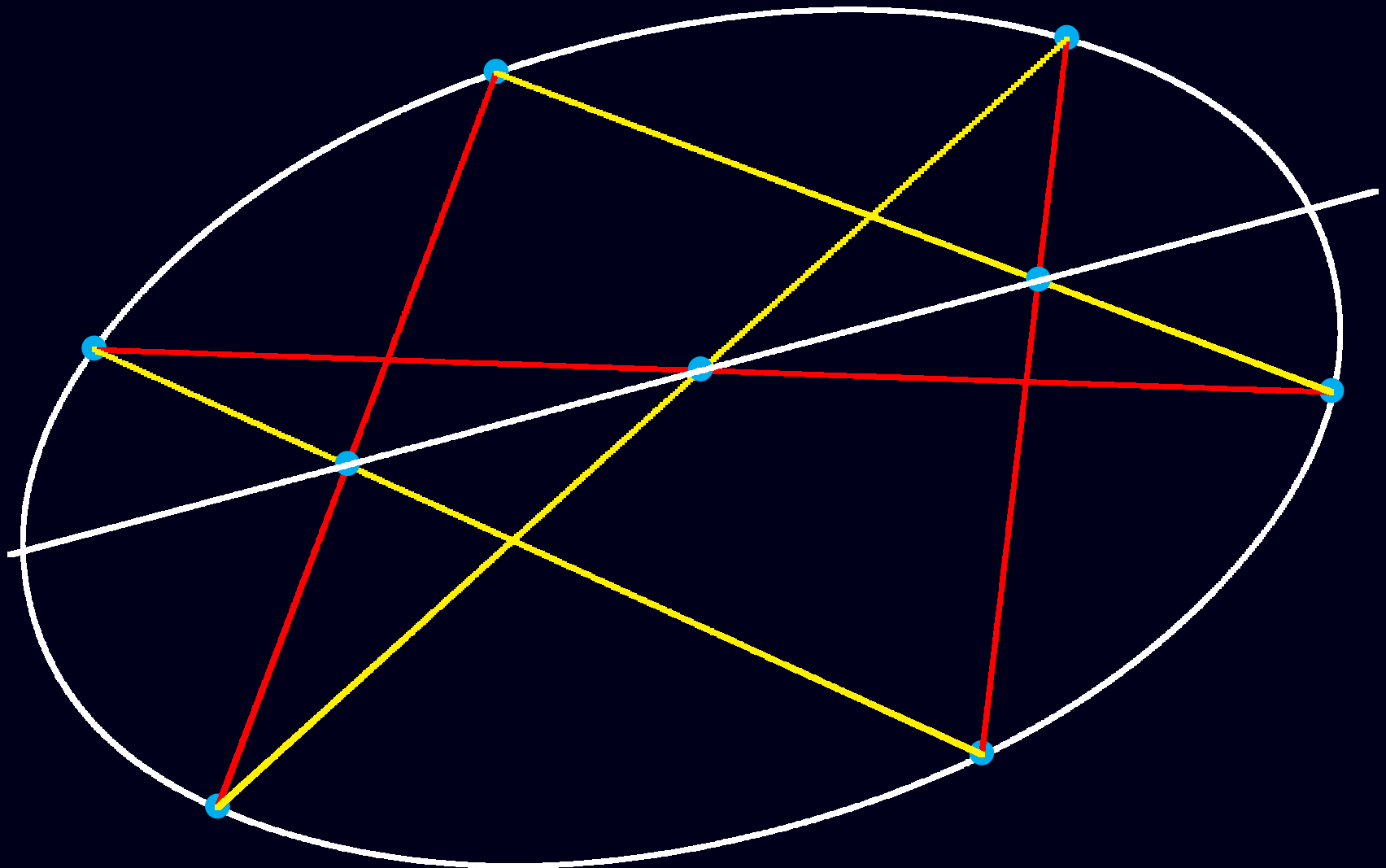
⋮

# Pappus' Theorem



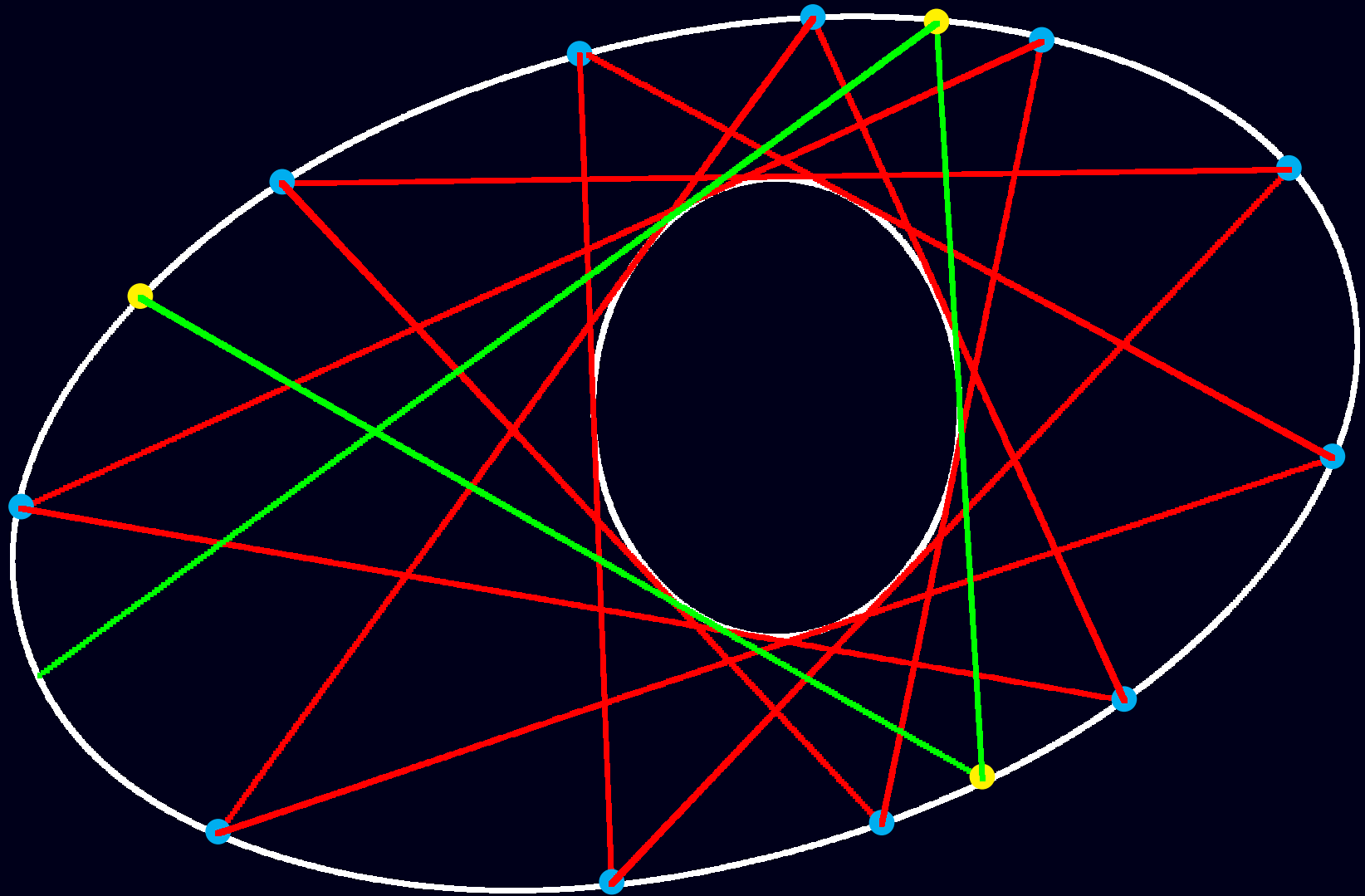
⋮

# Pascal's Theorem



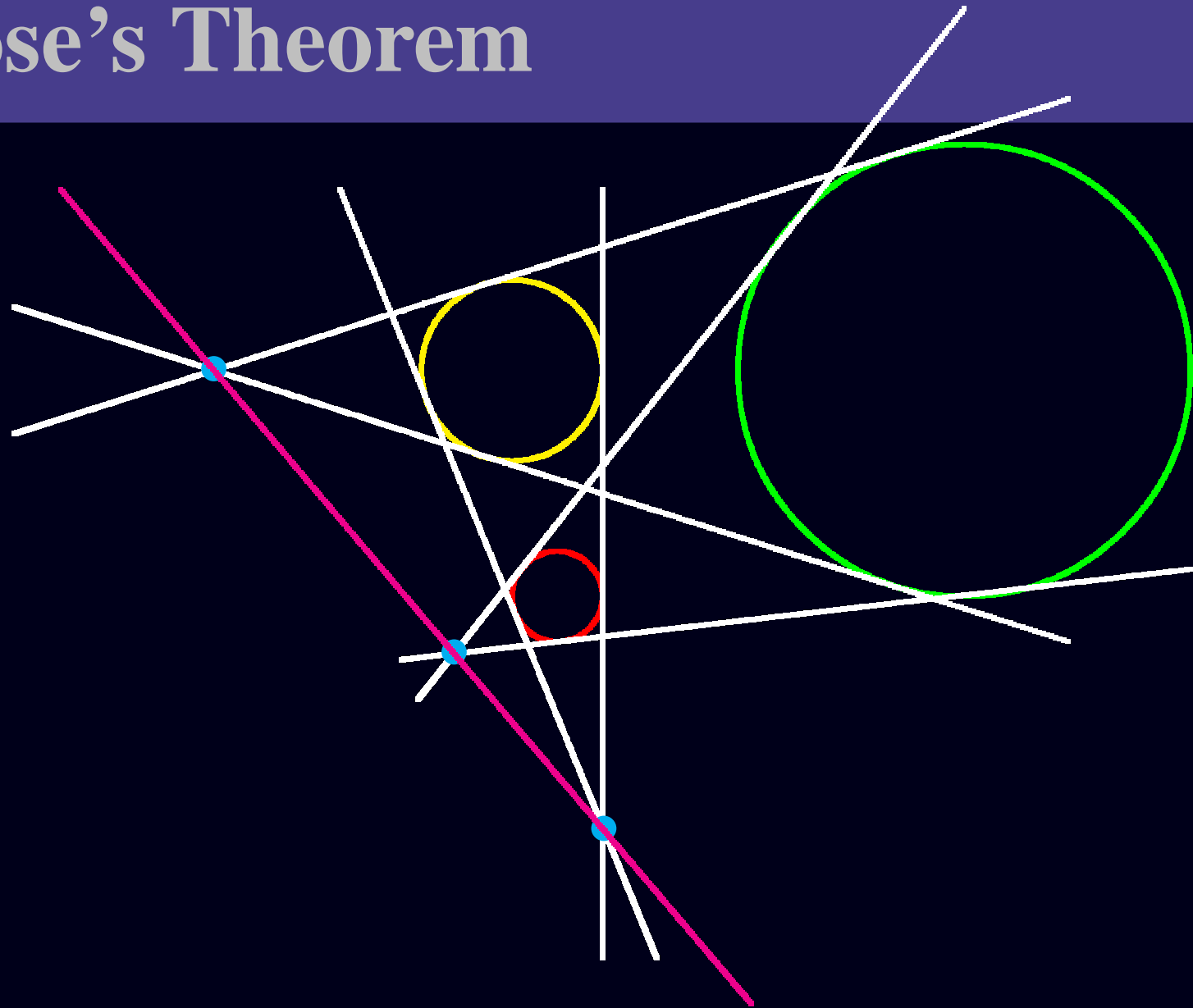
⋮

# Poncelet's Theorem



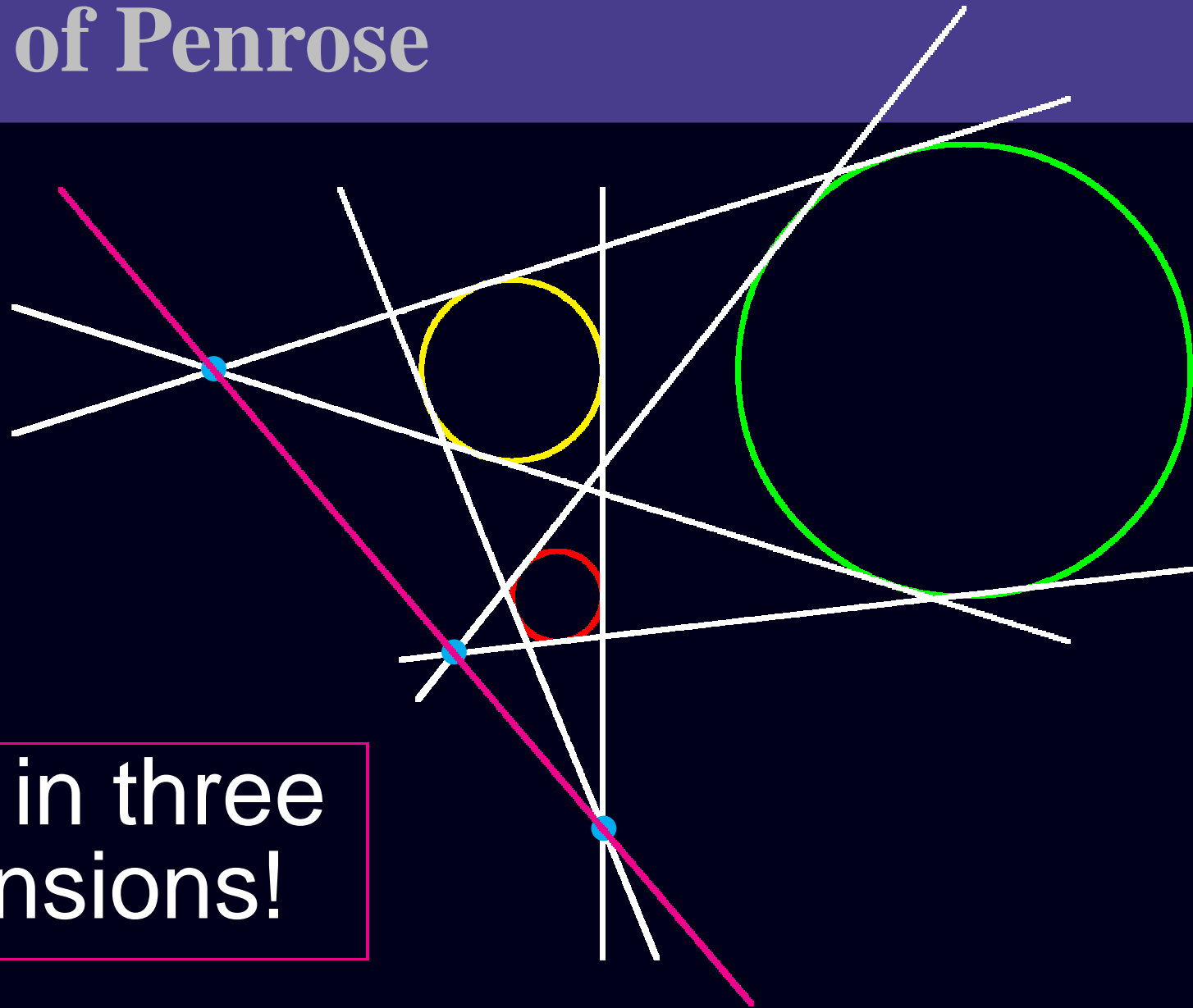
•  
•  
•

# Penrose's Theorem



•  
•  
•

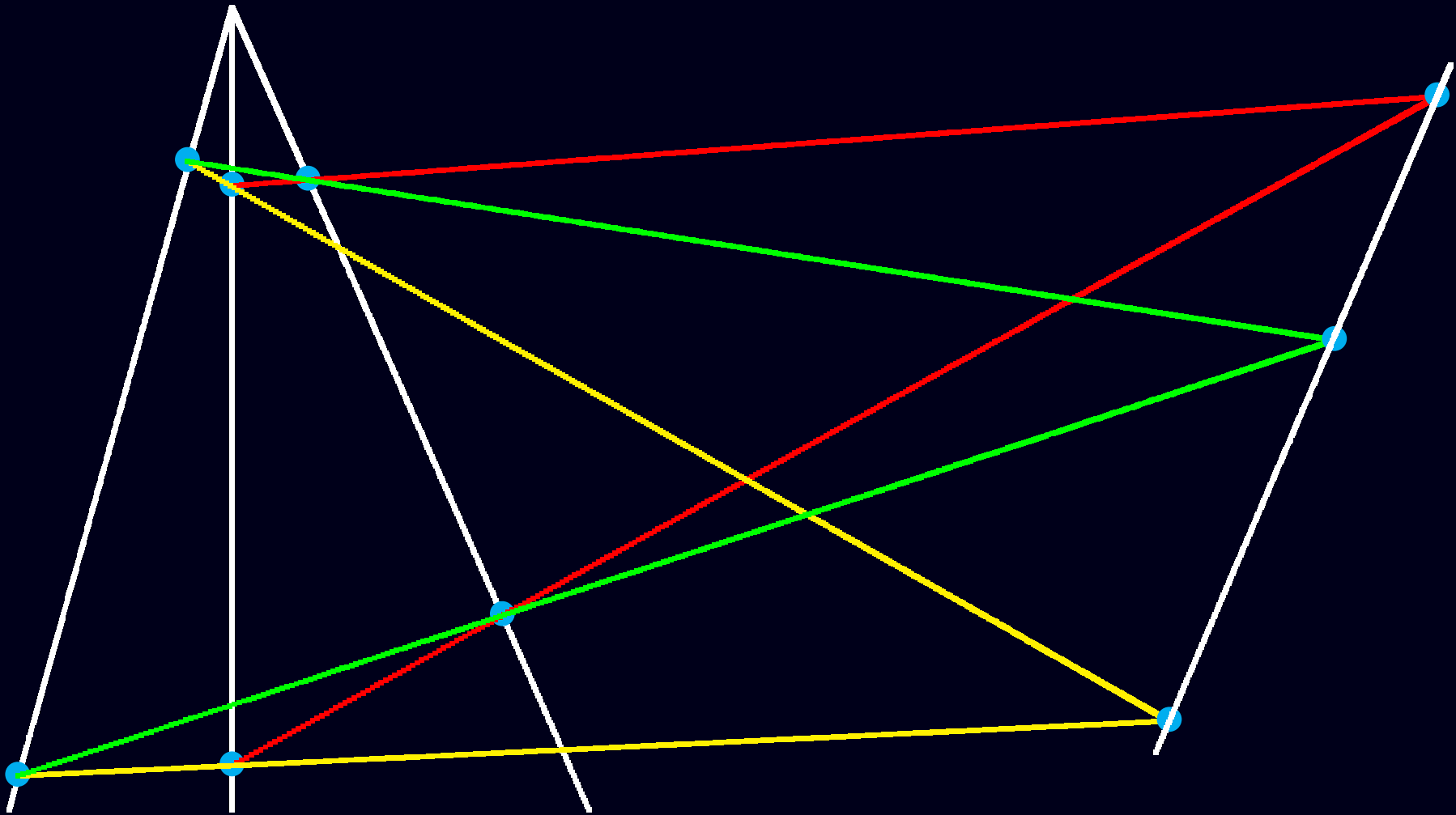
# Proof of Penrose



View in three dimensions!

⋮

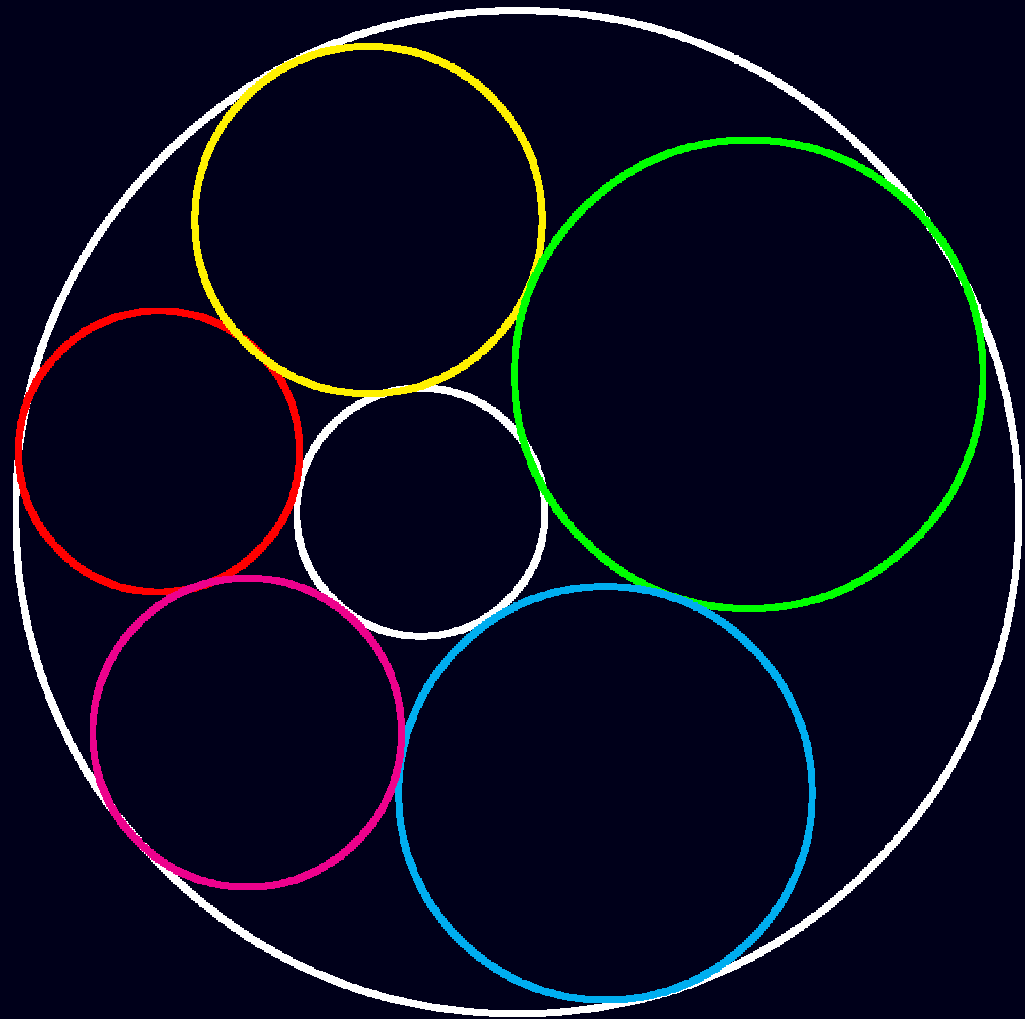
# Desargues' Theorem





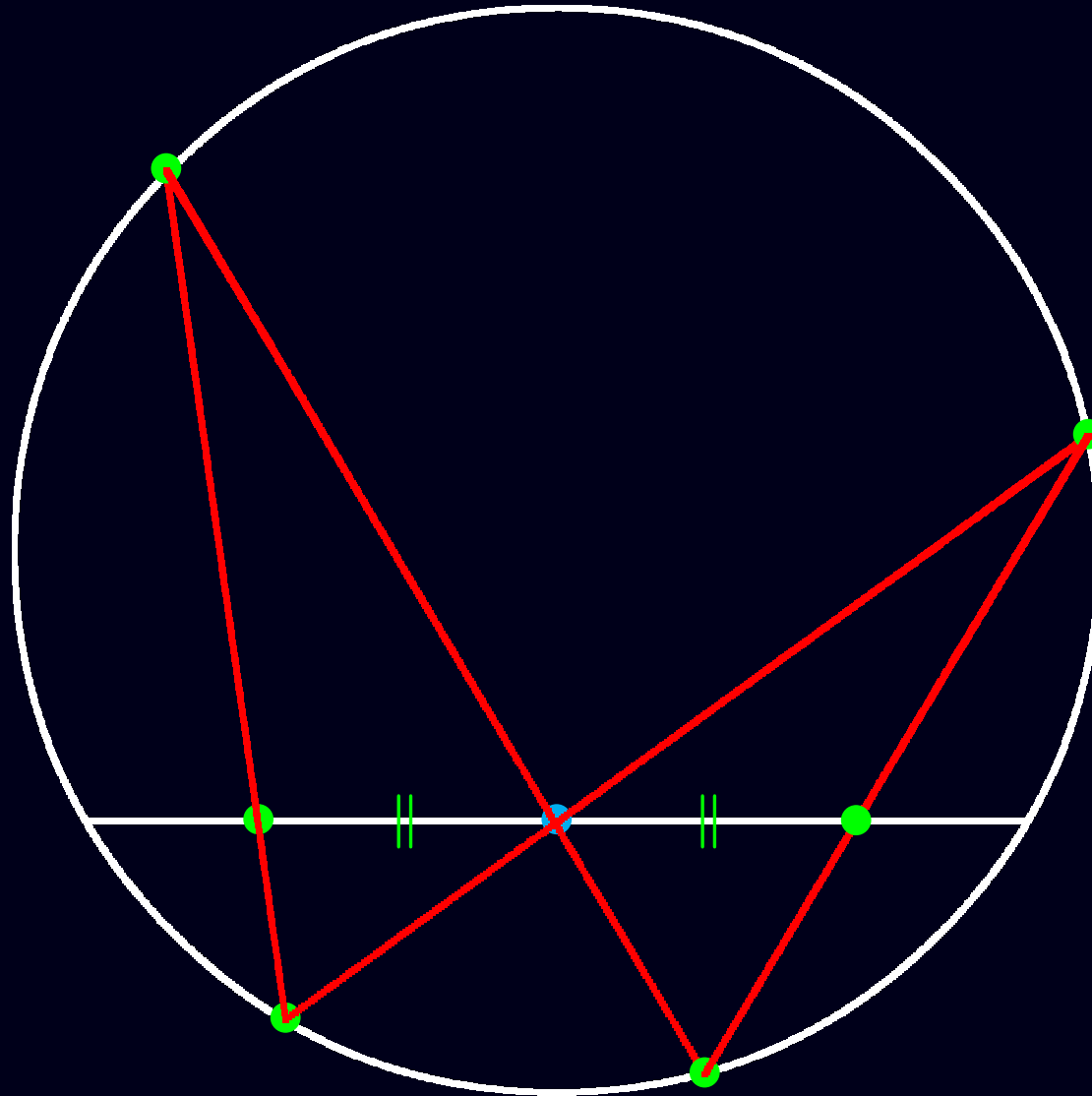
•  
•  
•

# Steiner's Porism



•  
•  
•

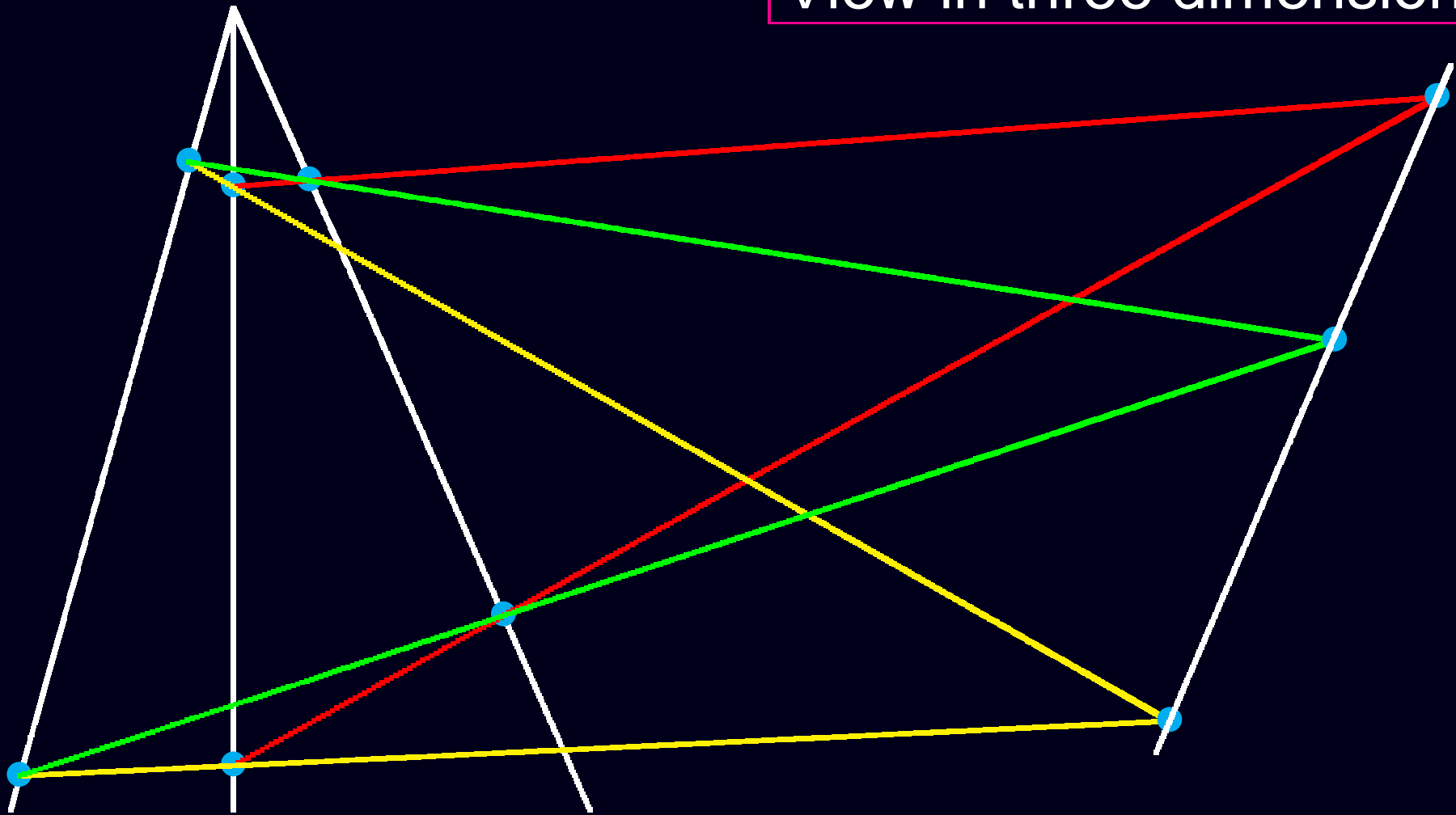
# Butterfly Theorem



⋮

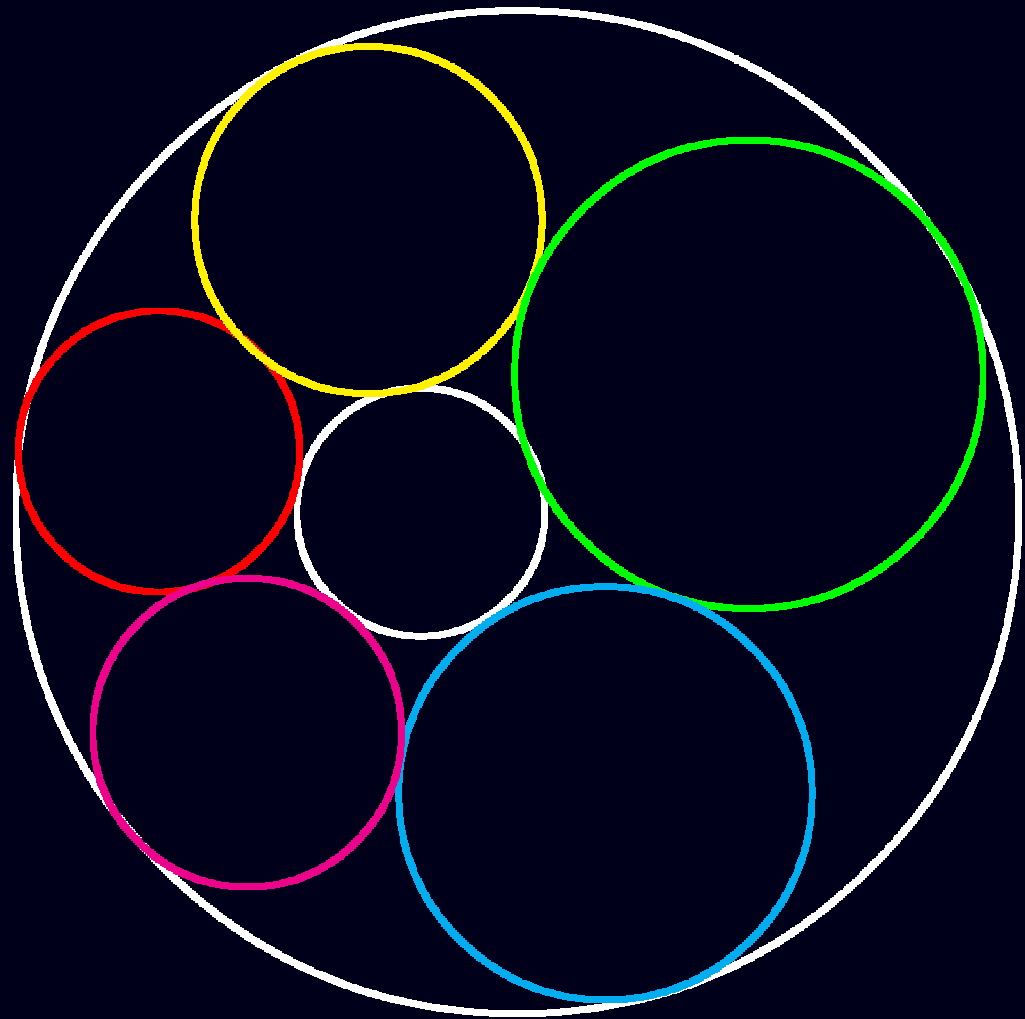
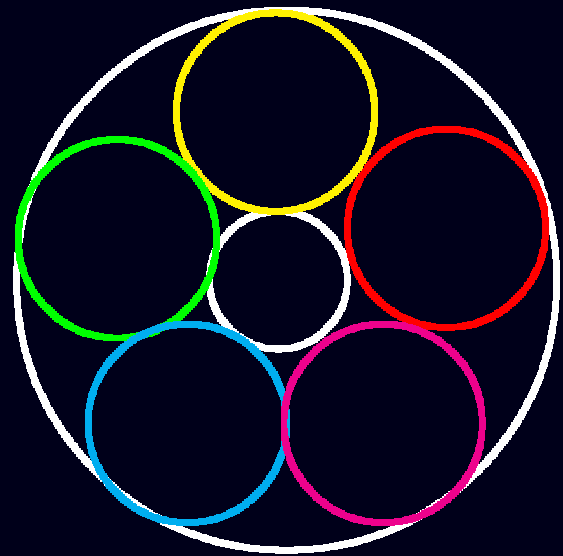
# Proof of Desargues

View in three dimensions!



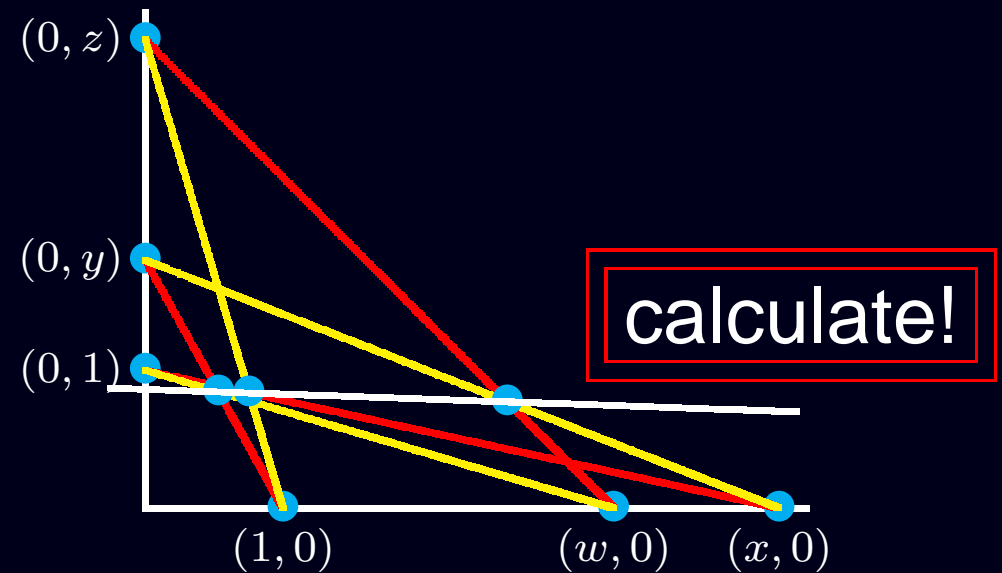
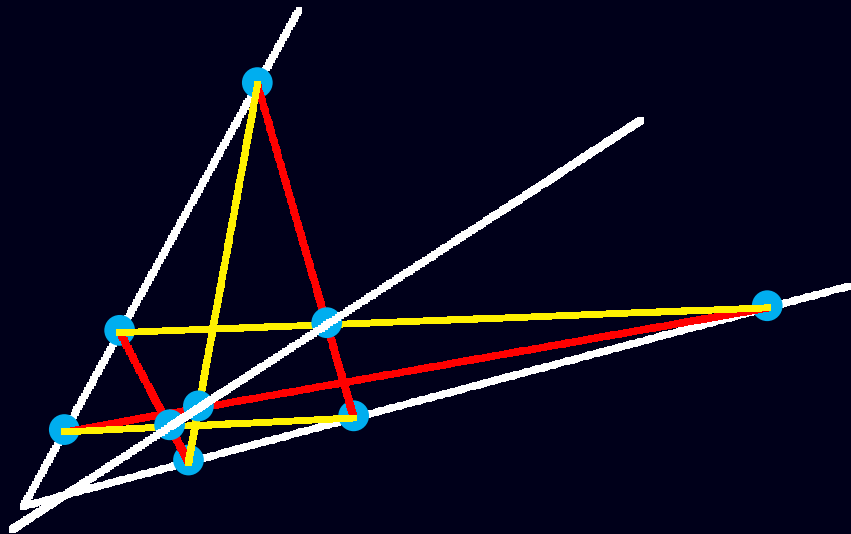
•  
•  
•

# Proof of Steiner



# Proof of Pappus

Brute force proof: normalise!



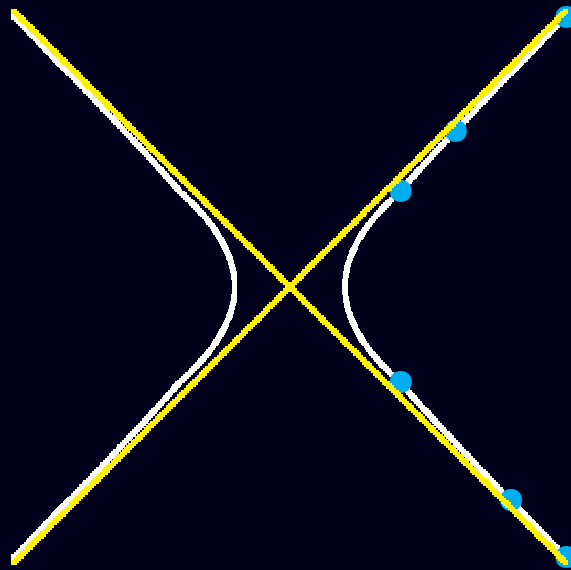
Elegant proof: deduce from Pascal!

# From Pascal to Pappus

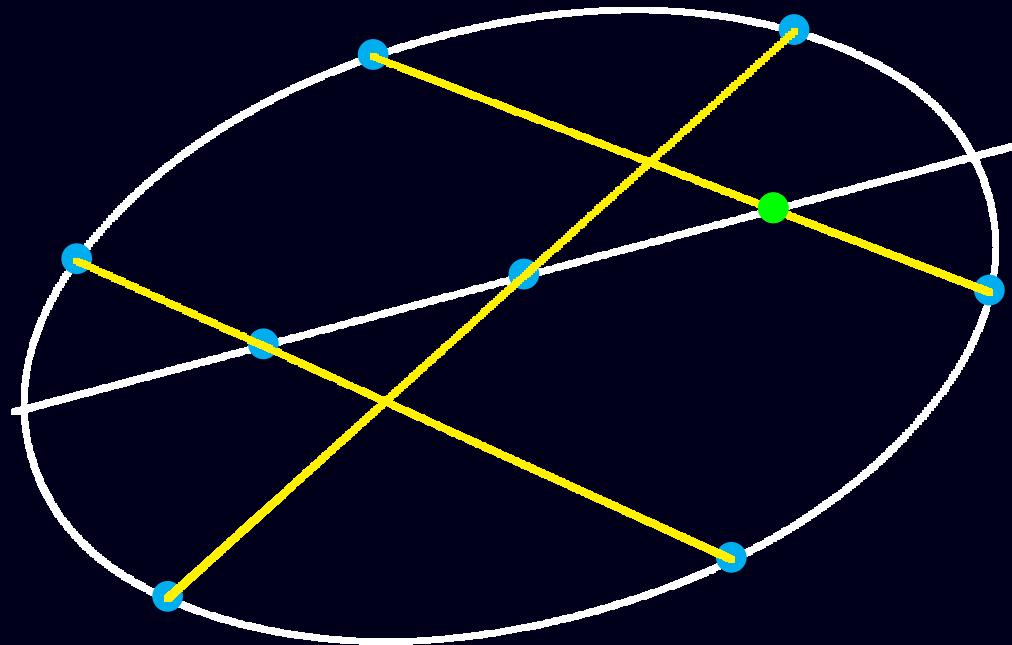
## Some algebra!

- ellipse  $x^2 + b^2y^2 = r^2$
- hyperbola  $x^2 - b^2y^2 = r^2$
- two lines  $(x - by)(x + by) = 0$

## Picture

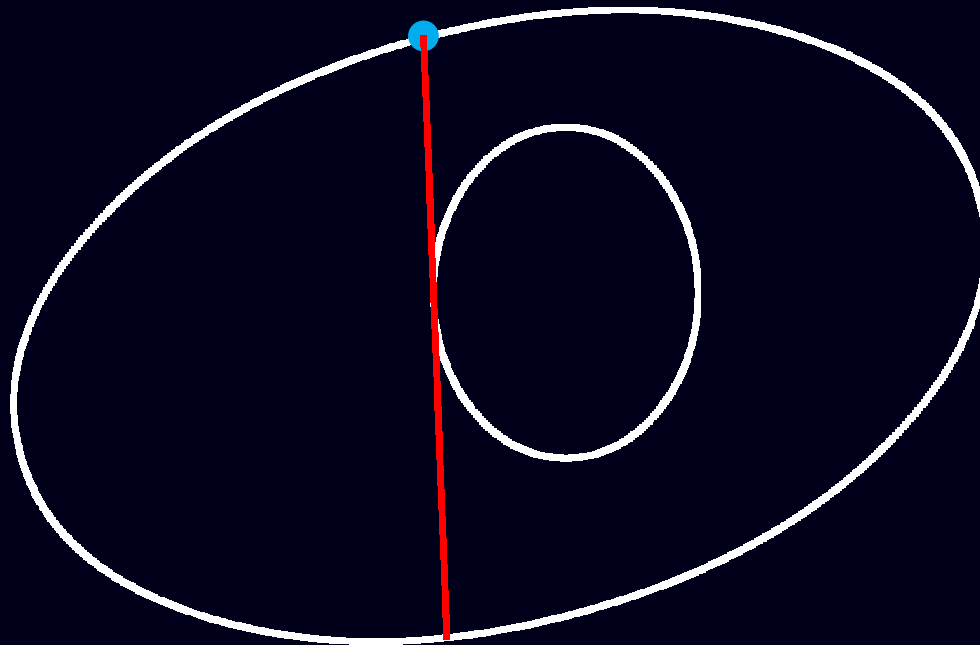


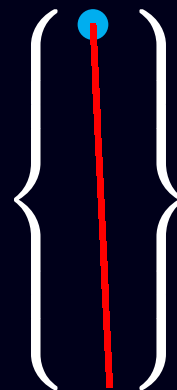
# Proof of Pascal

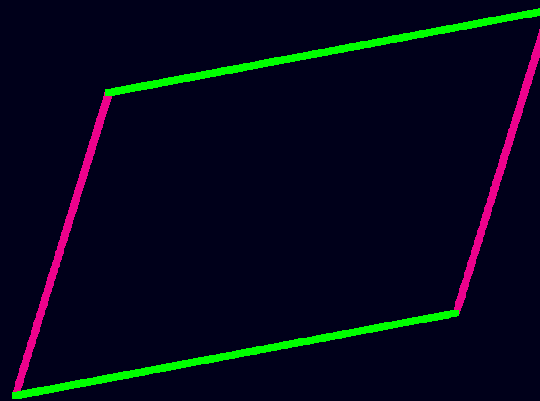


- Ellipse  $\cup$  Line  $\xrightarrow{\text{perturb}}$  Cubic!  $\cap$  Lines
- Use complex numbers!
- Add points at infinity!
- Doughnut  $\not\cong$  Ball: topology!

# Proof of Poncelet



 = Doughnut!

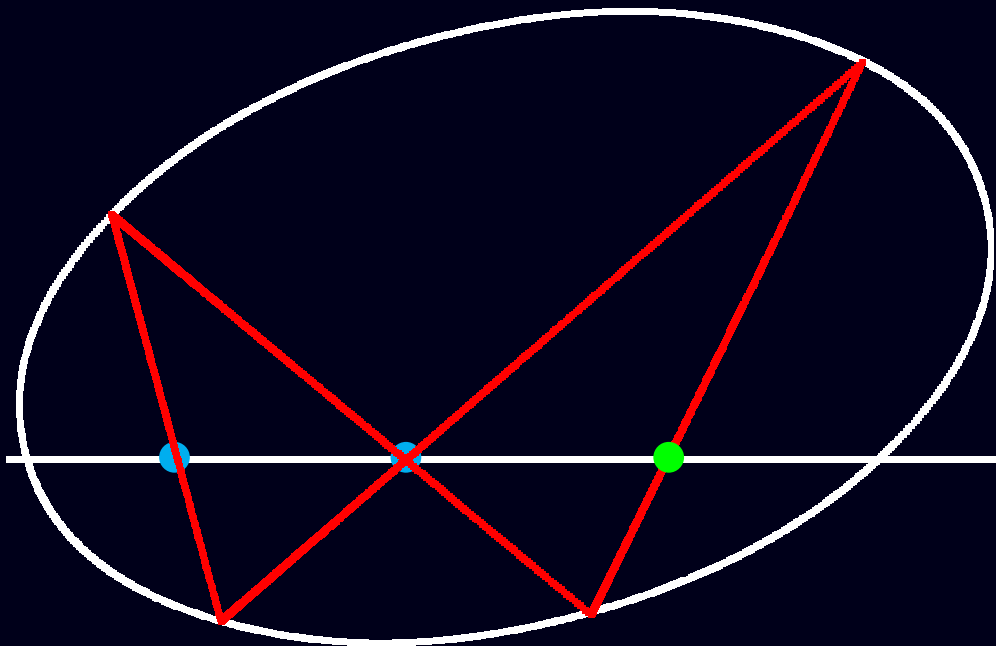


One  
Poncelet  
Step  
||  
Doughnut  
Translation



# Proof of Butterfly

Prove a more general theorem!



Ellipse  $\cup$  Line  
= Doughnut

Doughnut is  
a Group!

Colinearity  
 $\equiv$  Zero Sum!

# Further Reading

- <http://www.cut-the-knot.org/geometry.shtml>
- Leopold Flatto, Poncelet's Theorem, American Mathematical Society 2009

- 
- 
- 



고맙습니다



- 
- 
- 
- 
- 
- 
- 
- 
- 
- 
-