Prolongation 101
UoW 5/9/14

Algebra

M $n\times n$ matrix $M = \frac{1}{2}(M+M^t) + \frac{1}{2}(M-M^t)$ symm + skew

$$M_{ab} = \frac{1}{2}(M_{ab}+M_{ba}) + \frac{1}{2}(M_{ab}-M_{ba})$$

$$= M_{(ab)} + M_{[ab]} \quad \text{irreducible}$$

- $\partial_a X_b = \partial_{(a}X_{b)} + \partial_{[a}X_{b]}$

prolong ↑ (cf. Matthew's talk) — $\partial_{[a}X_{b]}$: curl (in $\mathbb{R}^3$) well understood

$\Lambda^0 \to \Lambda^1 \to \Lambda^2 \to \Lambda^3 \to$

$f \mapsto \partial_a f \quad F_{ab} \mapsto \partial_{[a}F_{bc]}$

$X_a \mapsto \partial_{[a}X_{b]}$

kernel and range ✓✓

⋮

$\partial_a X_b = F_{ab}$
$\partial_a F_{bc} = 0$ } closed

$X_a = s_a + m_{ab}x^b$ (translations: s_a; rotations: $m_{ab}x^b$)

- $M = M - \frac{1}{n}(\mathrm{tr}M)\mathbb{1} + \frac{1}{n}(\mathrm{tr}M)\mathbb{1}$

irreducible ⇝ divergence

⇝ $(\partial_a X^b)_\circ = 0$ prolong

$\partial_a X^b = \mu\,\delta_a{}^b$
$\nabla_a \mu = 0$ } closed

$X^a = s^a + m x^a$

- range of $X_b \mapsto \partial_{(a}X_{b)}$?

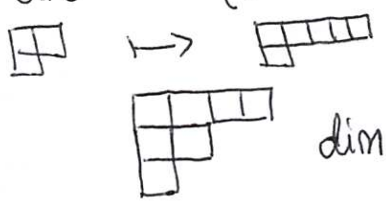
connection etc etc reeks of Riemannian differential geometry

diagram ← KEY

NB: In 3 dimensions ∃ mileage in numerical analysis!

Arnold - Falk - Winther

Ingredients – lotsa repⁿ thy! EG $\phi_{cde} \mapsto \partial_{(a}\partial_b\partial_c\phi_{d)ef}$



dim ≤ insert (cf. Liouville's theorem)

general theory
- in principle
- in practice

↑ art versus science

Grolitons too or G. Ricci

many more examples
- Killing (as above!)
- metrisability / mobility
- conf Killing ← $X_a = s_a + m_{ab}x^b + \lambda x_a + r^b x_b x^a - \frac{1}{2} r_a x^b x_b$ ← explain
- conf to Einstein
- Einstein-Weyl! (nonlinear)

general parabolic theory
Čap - Slovák - Souček...