$$\mathfrak{sl}(3,\mathbb{R}) \begin{cases} h = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} & x_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & y_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ H = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} & X = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} & Y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\ & x_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & y_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{cases}$$

$$\begin{aligned} \star\star\star \quad c_3 \equiv\ & 2h^3 - 2H^3 + 3h^2H - 3hH^2 - 18H^2 - 9hH - 18h - 36H \\ & - 9Hy_2x_2 + 9hy_2x_2 + 27y_2x_1X + 9hy_1x_1 + 18Hy_1x_1 \\ & - 18hYX - 9HYX + 27y_1Yx_2 - 54YX - 27y_2x_2 \end{aligned}$$

```
c3:=proc(f) 2*h(h(h(f)))-2*H(H(H(f)))+3*h(h(H(f)))-3*h(H(H(f)))
           -18*H(H(f))-9*h(H(f))-18*h(f)-36*H(f)
           -9*H(y2(x2(f)))+9*h(y2(x2(f)))+27*y2(x1(X(f)))
           +9*h(y1(x1(f)))+18*H(y1(x1(f)))-18*h(Y(X(f)))
           -9*H(Y(X(f)))+27*y1(Y(x2(f)))-54*Y(X(f))-27*y2(x2(f)) end;
h:=proc(f) u*diff(f,u)-v*diff(f,v) end;
x1:=proc(f) u*diff(f,v) end;
y1:=proc(f) v*diff(f,u) end;
H:=proc(f) v*diff(f,v)-w*diff(f,w) end;
X:=proc(f) v*diff(f,w) end;
Y:=proc(f) w*diff(f,v) end;
x2:=proc(f) u*diff(f,w) end;
y2:=proc(f) w*diff(f,u) end;
f:=u^7+2*v^2+6*u^4+exp(v*u^3)+u^8*v^15*w^49+v*exp(w)+w*cos(u*v);
simplify(x1(c3(f))-c3(x1(f)));
simplify(y1(c3(f))-c3(y1(f)));
simplify(h(c3(f))-c3(h(f)));
simplify(X(c3(f))-c3(X(f)));
simplify(Y(c3(f))-c3(Y(f)));
simplify(H(c3(f))-c3(H(f)));

cas2:=a^2+b^2+a*b+3*a+3*b;
cas3:=2*a^3-2*b^3+3*a^2*b-3*a*b^2-18*b^2-9*a*b-18*a-36*b;
eq2:=subs({a=u,b=v},cas2)-cas2;
eq3:=subs({a=u,b=v},cas3)-cas3;
solve({eq2,eq3},{u,v});

CAS2:=expand(subs({a=a-1,b=b-1},cas2));
CAS3:=expand(subs({a=a-1,b=b-1},cas3));
EQ2:=subs({a=u,b=v},CAS2)-CAS2;
EQ3:=subs({a=u,b=v},CAS3)-CAS3;
solve({EQ2,EQ3},{u,v});

assign(solve({a+b/2=A,3^(1/2)/2*b=B},{a,b}));
CAS2:=expand(CAS2);
CAS3:=expand(CAS3);

CAS2:=simplify(subs({A=cos(theta),B=sin(theta)},CAS2));
CAS3:=simplify(subs({A=cos(theta),B=sin(theta)},CAS3));
simplify(CAS3-2*(cos(3*theta)+9));
```

MikE, Nordfjordeid, 15th June 2015

$\star\star\star$ Jens Kroeske https://digital.library.adelaide.edu.au/dspace/handle/2440/49487