The projective models of Plucker line geometry and Lie sphere geometry - from the basic notions to Lie's line-sphere correspondence

Emanuel Huhnen-Venedey (Technische Universitat Berlin)

Date / Time / Location: Thu. January 19, 2012, 10:00 - 11:30, JD35.

Abstract:

According to Klein's Erlangen program, a geometry is a set of elements together with a group of transformations acting on them. 3-dimensional Plücker geometry is the geometry of lines in $\mathbb{R}P^3$, while 3-dimensional Lie geometry is the geometry of generalized oriented spheres in S^3. The set of generalized spheres includes points (spheres of vanishing radius) and planes (spheres of infinite radius). Both, lines in $\mathbb{R}P^3$ and spheres in S^3, each form a 4-dimensional manifold. In the projective models of the considered geometries, the corresponding elements (lines or spheres, respectively) are represented by points on a 4-dimensional quadric in $\mathbb{R}P^5$. The particular transformation group of the considered geometry becomes the subgroup of projective transformations of $\mathbb{R}P^5$ that preserve the corresponding quadric. In the lecture the basic concepts of Plücker geometry and Lie geometry will be explained and the projective models will be presented. Special attention will be given to the analogy of the two models, which manifests Lie's line-sphere correspondence.