Motion on increasing the Territories' members of the House of Representatives

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1 Executive Summary

1. Currently, each State and Territory is allocated a 'number' of Members in the House of Representatives in accordance to population; this is rounded to a whole number. For example, in 2003 the ACT was granted 2.42 Members; this was rounded down to 2.

2. This rounding affects smaller States and Territories more acutely since the amount rounded (in the above example, 0.42 was 'lost') is closer to the number of Members allocated. This leads to under-representation in the Territories.

3. Under current arrangements, Tasmania is over-represented, and this over-representation is 'more unfair' than the ACT's under-representation. Giving the Territories more Members (say, a minimum of 5 each to bring this in line with the States) is one way to redress this under-representation. But this swings the pendulum too far the other way: the Territories would be *over-represented*, and *much more* over-represented than Tasmania.

4. The alternative proposed here is to round up the number of Members given to the Territories so that their over-representation is *no more unfair than is Tasmania's.*

5. The formula included in the Motion below is Equation (2) on page 5.

Motion. *That the formula for determining the number of Members in the House of Representatives from the Australian Capital Territory and the Northern Territory be altered as follows. Find, amongst all of the States and Territories, the largest quotient of Members to quotas, where both Members and quotas are determined by the Constitution. Then the number of Members allocated to a Territory shall be the integer part of the product of this quotient and the Territory's population.*

2 Introduction

The House of Representatives is a directly representative house: the number of electorates allocated to a State/Territory is in direct proportion with the number of voters in that State/Territory. Thus, roughly, if one State has one-third of the nation's population, it should be allocated one-third of the members in the House of Representatives. To use an example from 2003: the population of the Commonwealth was estimated to be 19,205,190; the population of NSW was estimated to be 6,657,478. Thus, rounding[1] to the nearest thousand, NSW should receive $6657/19205 \approx 34.665\%$ of the electorates.

A problem quickly arises when we note that only an integer number of electorates can be allocated. It is impracticable[2] to ensure that each State/Territory is represented exactly in proportion. Rather, one wishes to ask the following question.

Question 1. *How should integer values (number of electorates) be assigned practicably to ratios (proportions of the total population), in the fairest way?*

The word 'practicably' needs special mention. The Constitution sets out a formula to determine the number of electorates for the *States*; this is examined in §3. Any attempt to change this formula would require a referendum. In particular, as per Section 128 of the Constitution, any attempt to *decrease* the number of electorates awarded to a *State* would not only require a referendum to pass (in a majority of States and with a majority of voters), but also with a majority of voters in *that particular State*. It is natural, therefore, that any practicable consideration should involve focussing, not on the States, but on the Territories.

Such consideration has twice been before Joint Standing Committees in [1] and [2]. Indeed, the terms of reference for [1] focus on

> The establishment of fixed formulae for determining the number of Senators and Members of the House of Representatives to which the Australian Capital Territory, the Northern Territory and other Territories are entitled.

This article will not focus on the Senate, nor on the proposals contained in [1] on 'other Territories'. Most of [1] is devoted to the practicability of legislating a *fixed* formula. Comparatively little attention has been placed on the nature of the formula itself, with the exception of Kirschbaum's [3] contribution, discussed in §4.2.1. This article proposes a formula in §4.

3 Current System

Section 24 of the Constitution gives a fairly straightforward formula to determine the number of electorates for a given State. The formula is designed to distribute electorates roughly in proportion to population, and to ensure that the number of members of the House "shall be, as nearly as practicable, twice the number of the senators."[3]

[1] There is bound to be an error in the population statistics of the States and Territories. It could be the case that this error is large enough such that a particular State/Territory *could* qualify for an extra electorate. This problem was considered by the Joint Standing Committee on Electoral Matters [2, p. xi], and is not considered in this article.

[2] If one were willing to accept the 2003 figures when rounded to the nearest thousand, one could, in principle, make a House of Representatives in which every State/Territory was represented *exactly* in proportion to its population. However, this would require a chamber with 19205 members.

[3] Here 'senators' means 'State senators' of which there are 72.

1. Suppose the population of the Commonwealth is C, and that each State S_i has population P_i. Form $q_i = 144P_i/C$.

2. These q_i's are the number of 'idealised' electorates that each State would be allocated. In general these q_i's will not be integers. By construction, the sum of these q_i's is 144, which is twice the number of State senators. Therefore ensuring that the number of Members of the House "shall be, as nearly as practicable, twice the number of the senators" is roughly equivalent to ensuring that each State's number of electorates as close to q_i.

3. The problem now is to assign an integer value to a given q_i. Let $q_i = [q_i] + \{q_i\}$, where $[q_i]$ denotes the integer part of q_i, and $\{q_i\}$ denotes the fractional part of q_i (or, equivalently, that part of q_i after the decimal point).[4] Given q_i we denote the number of electorates to be allocated by e_i. The Constitution provides a minimum of 5 electorates to each of the six States. The formula set forth in the Constitution is as follows.

   (a) If $\{q_i\} \leq \frac{1}{2}$, then $e_i = \max\{[q_i], 5\}$.
   (b) If $\{q_i\} > \frac{1}{2}$, then $e_i = \max\{[q_i] + 1, 5\}$.

   Thus, in (a) we take the larger of $[q_i]$ and 5, and, in (b) we take the larger of $[q_i] + 1$ and 5.

Section 122 in the Constitution allows Parliament to determine the number of electorates for the Territories. The Commonwealth Electoral Act 1918[5] does this by following Steps 1-3 above, with the slight alteration in Step 3: each of the Territories is provided with a minimum of one member. Thus, for a Territory

1. $\{q_i\} \leq \frac{1}{2}$, then $e_i = \max\{[q_i], 1\}$.

2. If $\{q_i\} > \frac{1}{2}$, then $e_i = \max\{[q_i] + 1, 1\} = [q_i] + 1$.

This formula can be readily applied to form the entries in Table 1, which is taken from Table 3.1 in [2]. The population of the Commonwealth, C, was 19,205,190.

Table 1: Electorates allocated in 2003

State/Territory	Population	Quotas	Electorates
New South Wales	6,657,478	49.9176	50
Victoria	4,888,243	36.6519	37
Queensland	3,729,123	27.9609	28
Western Australia	1,934,508	14.5049	15
South Australia	1,522,467	11.4154	11
Tasmania	473,371	3.5493	5
Australian Capital Territory	322,871	2.4209	2
Northern Territory	199,760	1.4978	1

It is insightful to compare the entries for South Australia and the Australian Capital Territory. Both were denied an extra electorate since the fractional parts of the quotas (0.4154 and 0.4209) did

[4] For example, if $q_i = 23.45$ then $[q_i] = 23$ and $\{q_i\} = 0.45$.

[5] This was augmented several times by acts altering the representation of each of the Territories. In 1990 all such acts were incorporated into the Commonwealth Electoral Act 1918.

not exceed one half. This can be viewed as South Australia and the Australian Capital Territory 'losing' 0.4154 and 0.4209 electorates. Even though these numbers are comparable, their size *relative* to the electorates awarded is not. The loss (equivalently, the under-representation) is more keenly felt by the Australian Capital Territory. We can measure this relative under-representation in the next section.

4 The proposed method: fairness measures

The *fairest method* would be for each State/Territory to be allocated q_i electorates. However, since q_i is rarely an integer, this is impossible. We should like to minimise the departure from the fairest method. Suppose a State/Territory, S_i, is allocated e_i electorates, and define

$$f_i = \frac{e_i}{q_i} - 1 \tag{1}$$

to be the *fairness measure* for S_i. In the fairest method, each $e_i = q_i$ and so the fairness measure is zero. When q_i is rounded up to $[q_i] + 1$ the State/Territory is over-represented, and when q_i is rounded down to $[q_i]$ the State/Territory is under-represented. We can now rephrase Question 1 to ask

Question 2. *Can we devise a model in which the f_i's are not too large?*

To attempt to answer Question 2 we use the data in Table 1 to compute the fairness measures: these are displayed in Table 2.

Table 2: Fairness measures in 2003

State/Territory	Quotas	Electorates	Fairness Measure
New South Wales	49.9176	50	0.0017
Victoria	36.6519	37	0.0095
Queensland	27.9609	28	0.0014
Western Australia	14.5049	15	0.0341
South Australia	11.4154	11	-0.0363
Tasmania	3.5493	5	0.4087
Australian Capital Territory	2.4209	2	-0.1739
Northern Territory	1.4978	1	-0.3324

Note that both of the Territories are under-represented.[6] The fairness measure for Tasmania is high (in fact, the highest) owing to its receiving a minimum of 5 electorates. Given that the likelihood of removing this minimum is extremely remote (see the last paragraph of §2), and given that any adjustment to the States' numbers would require a referendum, we henceforth consider only the Territories.

Suppose the maximum fairness measure from the States is $f_M = e_M/q_M - 1$. We aim at allowing the Territories more electorates, provided that their fairness measures do not exceed f_M. It follows that the Territories can be allocated e_i electorates, provided that $e_i \leq \frac{e_M q_i}{q_M}$. This can be achieved

[6] South Australia is also under-represented, but only slightly when compared with the Territories.

by choosing e_i to be the integer part of $\frac{e_M q_i}{q_M}$, that is

$$e_i = \left[\frac{e_M q_i}{q_M} \right]. \tag{2}$$

4.1 Example from 2003

The 2003 data in Table 2 indicate that Tasmania has the maximal fairness measure of 0.4087. Thus, using $e_M = 5$ and $q_M = 3.5493$ we can graph the right side of (2); we do this in Figure 1. We can now read off the number of electorates for the Territories given their quotas. The points at

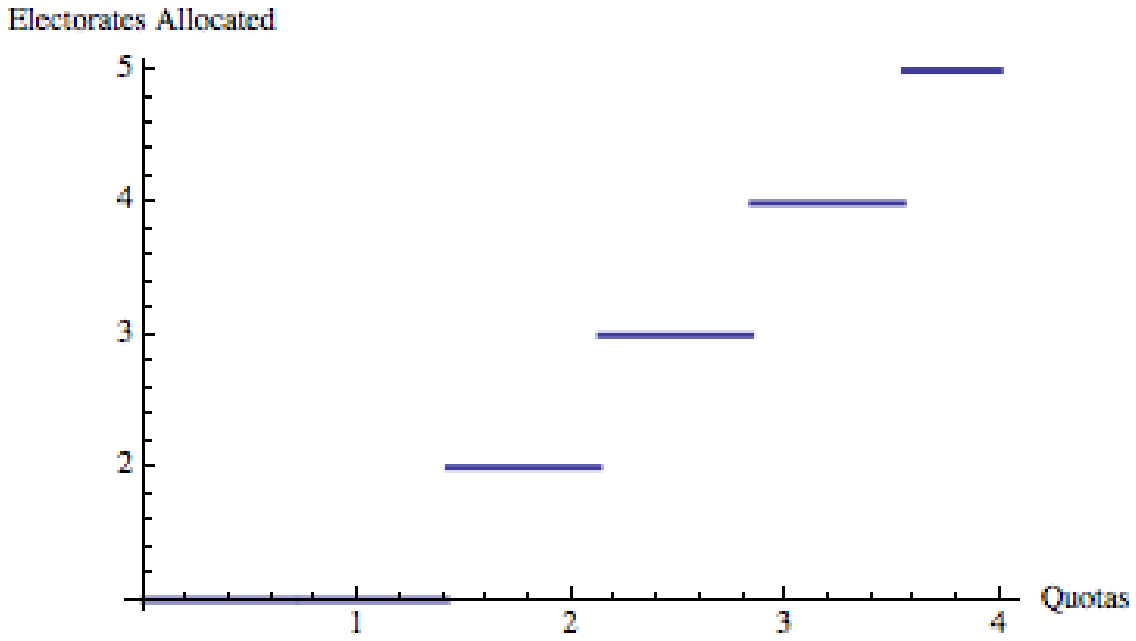

Figure 1: The jump in the number of electorates is given in (3).

which the number of electorates jumps by 1 are calculated by solving (2) for integer values of e_i. Indeed we deduce the following.

$$\begin{aligned} e_i &= 1, \quad \text{when } q_i \leq 1.4197 \\ e_i &= 2, \quad \text{when } 1.4198 \leq q_i \leq 2.1295 \\ e_i &= 3, \quad \text{when } 2.1296 \leq q_i \leq 2.8394 \\ e_i &= 4, \quad \text{when } 2.8395 \leq q_i \leq 3.5493 \\ e_i &= 5, \quad \text{when } 3.5494 \leq q_i \leq 5.5.^{7} \end{aligned} \tag{3}$$

Under this model, the 2003 results would have allocated the Northern Territory 2 electorates, and the Australian Capital Territory 3.

[7] Note that, once $q_i > 5.5$, we can revert to our existing model of rounding up whenever $\{q_i\} > \frac{1}{2}$. It should also be remarked that it is unlikely that either of the Territories would have more than 5 quotas.

4.2 Comparison with other models

It has been proposed[8] that it is unfair that the Territories are under-represented when Tasmania is over-represented. It has further been proposed that each of the Territories should be awarded a minimum number of electorates (larger than the current minimum of one)[9]. Not only does this article obviate the need for such arbitrary values, it also shows that these could lead to *greater* unfairness in distribution. For example, a minimum of 5 electorates for the Australian Capital Territory when fewer than 3.5494 quotas are obtained would lead to its having the highest fairness measure amongst all States and Territories.

4.2.1 The Kirschbaum Model

Miko Kirschbaum [3] submitted an alternative formula to the Joint Standing Committee in 2003. An analysis of his formula is given in Appendix A. Here we make three observations.

1. The model in this article agrees with Kirschbaum's model for many values of q_i.
2. Kirschbaum's model would allocate two electorates to a Territory with only 1.34 quotas. This could lead to *over*-representation of the Territory by giving it the largest fairness measure.
3. Although Kirschbaum's model often returns a fairness measure closer to zero than the model given in this paper, it often leads to under-representation.

In light of these points the model proposed in this article appears to be superior.

5 Conclusion

The proposed model in this article allocates electorates more fairly to the Territories. It is proposed that

1. Suppose the six States have q_i quotas, and, as a result of the formula given in the Constitution, are allocated e_i electorates.
2. Suppose the largest of the values $f_i = e_i/q_i - 1$ occurs at $i = M$.
3. Award each of the Territories $[e_M q_i/q_M]$ electorates, where $[x]$ denotes the integer part of x.

A Analysis of Kirschbaum's model

Kirschbaum's formula is, in the notation of this article, to allocate

$$e_i = \text{round}\left(\frac{q_i + \sqrt{q_i^2 + 1}}{2}\right) \tag{4}$$

[8] See, in particular, the impassioned dissenting report [1, pp. 55-58].

[9] In [2] it is remarked that awarding a minimum of 2 would almost be nugatory as far as the Australian Capital Territory is concerned.

electorates to a Territory with q_i quotas. That is, the quantity in (4) is rounded up if its fractional part is greater than $\frac{1}{2}$ and rounded down otherwise. The formula in (4) is appealing since since it leads to a low threshold for Territories with a low quota. Indeed, whenever $1.34 \leq q_i < 2.4$, according to (4) we award 2 electorates. This ought to be compared with the stipulation in (3), in which 2 electorates are awarded when $1.4198 \leq q_i \leq 2.1295$. In Figure 2 Kirschbaum's model is plotted against the model in this article.

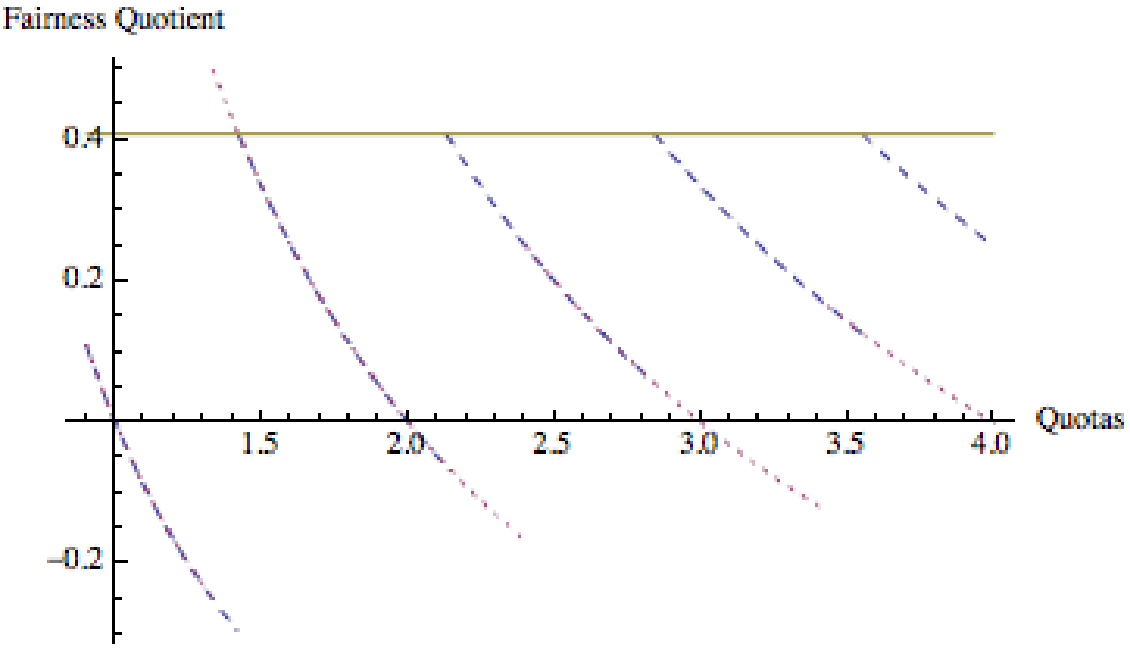

Figure 2: The horizontal line is $f_M = 0.4087$; Kirschbaum's model is red and dotted; the model in this article is blue and dashed.

References

[1] *Determining the Entitlement of Federal Territories and New States to Representation in the Commonwealth Parliament*, Joint Standing Committee on Electoral Reform, 1985.

[2] *Territory Representation*, Joint Standing Committee on Electoral Matters, 2003.

[3] Kirschbaum, M. *Territory Representation: Submission number 7*, Joint Standing Committee on Electoral Matters, 2003.