Mean curvature equation and mean curvature flow

The mean curvature equation

We found a simple and elementary proof for the interior gradient estimate for the mean curvature equation [mc1]. This proof also applies to the mean curvature flow of graphs. (Once the gradient is bounded, higher regularity follows from the regularity theory for elliptic equations)

In [mc2] we proved a variant of the Harnack inequality for the prescribing mean curvature equation (the Harnack inequality in the classical sense does not hold for the mean curvature equation), by which we also proved the weak convergence of the mean curvature measure.

The mean curvature flow

Let $\mathcal{F} = \{F_t : \ t \in [0, T)\}$ be a mean curvature flow in $\mathbb{R}^{n+1}$ which develops the first singularity at time T.

Theorem 1. Assume F_0 is a closed hypersurface with positive mean curvature. Then at the first time singularities, any blow-up sequence converges locally smoothly along a subsequence to a noncollapsing convex blow-up solution.

This theorem was proved by White using the geometric measure theory. In [mc4] we found a different proof by using Huisken and Sinestrari's curvature pinching estimate and some basic a priori estimates for parabolic equations. Another different proof was later discovered by Ben Andrews.

Remark. The positive mean curvature condition is necessary. Indeed, a torus can be a self-similar solution to the mean curvature flow.

To classify the blow-up solutions, we have [mc3]

Theorem 2.
(i) A blow-up translating solution in $\mathbb{R}^3$ is rotationally symmetric.
(ii) There exists non-rotationally symmetric translating solution in $\mathbb{R}^{n+1}$ for $n > 2$.
(iii) The blow-down of a blow-up solution is a shrinking sphere or cylinder.

Similar results for 2-convex (namely the least two principal curvatures $\kappa_1 + \kappa_2 > 0$) mean curvature flow have also been obtained by Huisken and Sinestrari.

References

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[mc4] W.M. Sheng, X.J. Wang, Singularity profile in the mean curvature flow, Methods and Applications of Analysis, to appear.