AN ASYMPTOTIC EXPANSION INSPIRED BY RAMANUJAN

RICHARD P. BRENT

Abstract

Corollary 2, Entry 9, Chapter 4 of Ramanujan's first notebook [1] claims that

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{nk} \left(\frac{x^k}{k!}\right)^n \sim \ln x + \gamma$$

as $x \to \infty$. This is known to be correct for the case $n = 1$, but incorrect for $n \geq 3$. We show that the result is correct for $n = 2$. We also consider the order of the error term, and discuss a different, correct generalisation of the case $n = 1$.

Comments

Only the Abstract is given here. The full report appeared as [2]. The results were also presented and appeared in [3]. The generalisation mentioned in the Abstract was suggested by Brent and McMillan in [4].

References

[1] B. C. Berndt, *Ramanujan's Notebooks, Part I*, Springer-Verlag, New York, 1985.

[2] R. P. Brent *An asymptotic expansion inspired by Ramanujan*, Technical Report CMA-MR02-93/SMS-10-93, CMA, ANU, February 1993, 7 pp. Also appeared in *Australian Math. Soc. Gazette* 20 (1993), 149-155. rpb139tr.

[3] R. P. Brent "Ramanujan and Euler's constant", presented at *Mathematics of Computation 1943–1993*, Vancouver, 9–13 August 1993. Appeared in *Proceedings of Symposia in Applied Mathematics* 48 (1994), 541-545. rpb139.

[4] R. P. Brent and E. M. McMillan, "Some new algorithms for high-precision computation of Euler's constant", *Math. Comp.* 34 (1980), 305–312. MR 82g:10002. rpb049.

Computer Sciences Laboratory, Australian National University, Canberra, ACT 0200

E-mail address: `rpb@cslab.anu.edu.au`

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