Jonathan Borwein 1951–2016: Life and Legacy

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Summary

This talk is about the Borwein family of mathematicians. We concentrate on Jonathan (Jon) Borwein (1951–2016), who spent his last years (2009–2016) as a Laureate Professor and founding Directory of CARMA at the University of Newcastle.

We also mention some of Jon's joint work with his father David Borwein (1924–2021) and younger brother Peter Borwein (1953–2020).

Families of mathematicians

There are several remarkable families of mathematicians.
It is hard to beat the Bernoullis: Jacob (1655-1705, also 1759–1789), Johann (1667–1748, 1710–1790, 1744–1807), Nicolaus (1687–1759, 1695–1726), and Daniel (1700–1782).

Closer to home, there are the Neumanns: Bernhard (1909–2002) and Hanna (1914–1971), and their sons Peter (1940–2020) and Walter (1946–).

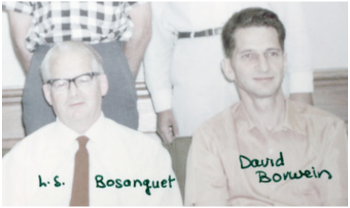
This talk is about the Borweins: David (1924–2021) and his sons Jonathan (1951–2016) and Peter (1953–2020).

I shall concentrate on Jonathan (Jon) Borwein, because of his Australian connection and because Jon was the only one of the three that I knew personally. However, I shall mention some of the joint work that David and Peter did with Jon.

The Borwein family of mathematicians

The story starts with David Borwein, who was born in Lithuania in 1924. but moved with his family to South Africa at the age of six (in 1930). In South Africa he met his future wife Bessie (later a Professor of Anatomy).

In 1948, David and Bessie moved to the UK, where David obtained a PhD (London) on a topic related to Cesàro summability, under the supervision of L. S. Bosanquet (an analyst and former student of G. H. Hardy).



Hardy → Bosanquet → David Borwein

The next generation: Jon and Peter

After David obtained his PhD, David and Bessie moved to Scotland, where David took up a lectureship at St Andrews. There Jon was born in 1951, and Peter in 1953.

Peter had "fond memories of growing up with Jon, as young boys in St Andrews". He mentioned fishing, looking after guinea pigs, and that he could

> *"keep Jon awake at night, by reminding him that the universe was infinite. This bothered him no end."* [1]

Jon and Peter both ended up as mathematicians, although Jon almost majored in History. He decided on mathematics in 1968. Peter said that he was not interested in mathematics until the second year of university – perhaps this was in reaction to his father's and brother's choice of mathematics as a career.

[1] Whether the universe is finite or infinite seems to be a question for cosmologists, not mathematicians.

St Andrews, 1957

Jon, David, and Peter Borwein, 1957

Learning probability theory by experiment?

Peter and Jon playing poker, 1959

An early example of Experimental Mathematics?

Jon and Peter fifty years later

Jon and Peter Borwein, Simon Fraser University, Canada, on the occasion of Peter's 55th birthday party, May 2008.
Thanks to Michael Coons for information on this photo.

On sibling rivalry (or lack of it)

There seems to have been no mathematical rivalry in the Borwein family (quite unlike the Bernoulli family!). Here are some remarks about Jon by Peter Borwein:

> *"We co-authored over 25 papers and books, and he solo wrote over 200 . . . He could accomplish in a day or two what took other people weeks . . . We worked together for over 35 years; we were in the same department for about 20 . . . I owe much of my career to him, because people thought they were getting him when they hired me.*
>
> *People thought we ought to compete, that there ought to be sibling rivalry. In fact there really was no rivalry . . . he was always generous with ideas, and with acknowledgment, and with giving credit, not just to me, but to his graduate students, and colleagues – who he could drive very hard, but if they could keep up with him the pay-off was rewarding."*

Jon's mathematical career – early years

The Borwein family moved to Ontario, Canada, in 1963. David and Peter stayed in Canada from then on. I don't have time to say more about their careers today (if you are interested, see Wikipedia).

Jon graduated with a B.A. (Hons. Math.) from the University of Western Ontario in 1971, and won a Rhodes scholarship, which allowed him to return to the UK. He obtained a D.Phil (Jesus College, Oxford) in 1974, on a topic in Optimisation, an area in which he would later become well known, but by no means his only interest.

Jon's career – later years

After his D.Phil in Oxford, Jon moved back to North America, where he held various positions, including at CMU, Waterloo, and Simon Fraser. At the end of his time in Canada, he held a Canada Research Chair at Dalhousie (2004–2009). Jon was president of the Canadian Mathematical Society 2000–2002, following in the footsteps of David, who held the same position in 1985–1987.

In 2009, Jon and Judith moved, with daughters Naomi and Tova, to Newcastle, where he became Laureate Professor of Mathematics and founding Director of the Priority Research Centre CARMA. I shall say more about CARMA later.
First, let's discuss some of Jon's research. For more details and references, see my recent paper on Jon in the new journal (itself partly inspired by Jon) *Maple Transactions*.

The Barzilai-Borwein algorithm

Suppose that we want to approximate a stationary point of a function $F(x)$ that is differentiable in the neighbourhood of a starting point $x_0 \in \mathbb{R}^n$. Several optimisation methods are based on the iteration

$$x_{k+1} = x_k - \gamma_k \nabla F(x_k), \; k \geqslant 0,$$

but differ in their choice of "step sizes" γ_k. One choice used in the 1988 Barzilai-Borwein paper is

$$\gamma_k = \frac{(x_k - x_{k-1})^T (\nabla F(x_k) - \nabla F(x_{k-1}))}{||\nabla F(x_k) - \nabla F(x_{k-1})||_2^2} .$$

The motivation for this choice is that it provides a two-point approximation to the secant equation underlying quasi-Newton methods. This generally gives much faster convergence than the classical method of steepest descent, while having comparable cost per iteration and storage requirements.

Experimental mathematics

Jon was a great advocate of *experimental mathematics*. He used computation both to discover new mathematics and to suggest, prove, or disprove, various interesting conjectures. Many examples are given in his books on the subject (2004–2008) with co-authors David Bailey, Keith Devlin, and others.

Some mathematicians dislike the name "experimental mathematics". An alternatives is "mathematics informed by computation", as for a workshop to be held in Park City, Utah, July-Aug. 2022.

When Jon established a Research Centre in Newcastle, he avoided "Experimental" and used the phrase "Computer Assisted Research Mathematics", perhaps influenced by the resulting acronym CARMA.

An extension of Jon's vision is described in a recent paper by Geordie Williamson, Demis Hassabis et al., "Advancing mathematics by guiding human intuition with AI".

The surprising sinc function

To give an example that appeared in a 2008 joint paper with Robert Baillie and David Borwein, consider the question: for which positive integers N does

$$\frac{1}{2}+\sum_{n=1}^{\infty}\prod_{k=0}^{N}\operatorname{sinc}\left(\frac{n}{2k+1}\right)=\int_0^{\infty}\prod_{k=0}^{N}\operatorname{sinc}\left(\frac{x}{2k+1}\right)\mathrm{d}x\ ? \quad (1)$$

(here $\operatorname{sinc}(x)=\sin(x)/x$ if $x\neq 0$, and 1 if $x=0$). Experimentation with Sage/Magma/Maple/Mathematica suggests that (**??**) is an identity for $1\leqslant N\leqslant 6$. However, in a warning about extrapolating results, Jon showed that (**??**) holds for $1\leqslant N\leqslant 40248$, but fails for all $N>40248$. The proof of (**??**) depends on the assumption that $\sum_{n=0}^{N}1/(2n+1)\leqslant 2\pi$, which is false for $N>40248$.

Jon and π

Jon was fascinated by the transcendental constant

$$\pi = 3.14159265358979\cdots,$$

and gave many fascinating talks on π, often associated with an annual celebration of "pi day" on March 14th (US-style date).

Indeed, a talk on π is a good way to introduce some interesting mathematics to a general audience with a mathematical background, or to undergraduate students. Mathematical topics that can be motivated by π include the concepts of irrational and transcendental numbers, rates of convergence of series and algorithms, normality of the digits of π in decimal or binary (still an open problem), and Euler's famous relation $e^{i\pi} = -1$ that connects the constants $e, \pi, i,$ and -1.

Linearly convergent algorithms for π

Most algorithms for the computation of π converge *linearly*, i.e. the number of correct digits increases approximately linearly with the number of iterations or the number of terms summed in a series.

For example, this applies to Ramanujan's famous formula

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 \, 396^{4n}}$$

and similar formulas by the Chudnovsky brothers and others.

More linearly convergent algorithms

Similarly, the Bailey-Borwein-Plouffe (BBP) formula

$$\pi = \sum_{n=0}^{\infty} 2^{-4n} \left(\frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6} \right)$$

converges linearly.

The “Borwein” here is Peter Borwein. The formula was discovered by Simon Plouffe and published in a joint paper. It allows us to efficiently compute selected digits in the (binary) representation of π. No such formula is known for base ten.

The arithmetic-geometric mean (AGM)

Given two real positive numbers a_0 and b_0, define

$$a_{n+1} = (a_n + b_n)/2, \quad b_{n+1} = (a_n b_n)^{1/2}.$$

Then a_n and b_n tend to a common limit $M(a_0, b_0)$, called the *arithmetic-geometric mean* or AGM, that was first studied by Lagrange in the late 18th century, and slightly later (independently) by Gauss.

Gauss showed that the AGM can be expressed using complete elliptic integrals of the first kind:

$$\frac{\pi}{2M(a,b)} = \int_0^{\pi/2} (a^2 \cos^2 \phi + b^2 \sin^2 \phi)^{-1/2} \, d\phi \, .$$

A critical property is that the *error term* $\varepsilon_n := a_n - M(a_0, b_0)$ converges *quadratically* to zero: $\varepsilon_{n+1} = O\left(\varepsilon_n^2\right)$.

Superlinearly convergent algorithms for π

In the mid-seventies it was discovered (by RPB and Eugene Salamin, independently), that there exist *superlinearly convergent* algorithms for π (and for the elementary functions exp, ln, sin, arctan, etc., but that is another story). These algorithms depend on the AGM, and inherit the property that the number of correct digits increases geometrically rather than linearly. The first (and perhaps simplest) such algorithm is often called *Algorithm GL* after Gauss and Legendre.

For example, using eight iterations of Algorithm GL, the error in the computed approximation to π is less than 10^{-690}.

In contrast, the classical method of Archimedes, using inscribed and circumscribed polygons, gets only two correct bits per iteration, so requires more than one thousand iterations to obtain the same accuracy (even though the iterations are of comparable complexity).

Pi and the AGM

In 1987, Jon and Peter Borwein published their classic book *Pi and the AGM: A Study in Analytic Number Theory and Computational Complexity*. They comprehensively studied superlinearly convergent algorithms for π and generalisations to the fast computation of elementary functions, and much more, including (from chapter headings):

- Complete elliptic integrals and the AGM
- Theta functions, the AGM, and algorithms for π
- Jacobi's triple product, theta functions, . . . , applications
- Modular equations, . . . , algebraic approximations to π
- The complexity of calculating algebraic functions
- The complexity of π and the elementary functions
- General means and mean iterations
- Other approaches to the elementary functions
- The story of π, computation and transcendence

Pseudo-mathematics and financial charlatanism

Although Jon was primarily a pure mathematician, his interests extended much further and included aspects of applied mathematics and statistics. This can be illustrated by his work on mathematical finance. A significant contribution is his 2014 paper with Bailey, de Prado and Zhu, provocatively titled "Pseudo-mathematics and financial charlatanism".

The paper grew out of the authors' concern that, although mathematics had become a standard language to quantify financial phenomena, it was often used in a misguided fashion.

The paper demonstrated that many financial strategies and fund designs, claimed to be backed by extensive "backtests" (analyses based on historical market data), were nothing more than illusory artifacts resulting from statistical overfitting. The conclusion was that backtest overfitting is a likely reason why so many financial strategies and fund designs fail, despite looking good on paper.

Mathematical education

Jon had a passion for sharing his joy of mathematics with students and a more general audience. Some of his research topics were particularly well-suited for communication to high school and undergraduate students. We have already mentioned his interest in experimental mathematics and his "pi-day" activities.

Jon was a keen blogger, starting this activity in 2009 when he and David Bailey founded the Math Scholar blog, which now has over 200 articles on a wide range of topics, covering many facets of modern mathematics, computing, and science. The blog is still running, thanks to David.

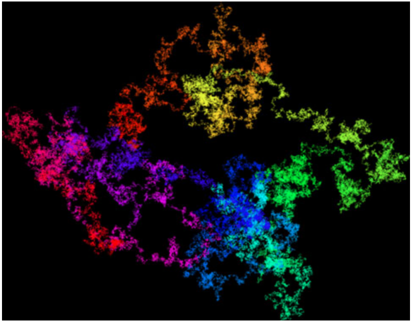
Visualisation

Jon regarded visualisation as a powerful tool for both experimental mathematicians and mathematical communicators. Examples may be found in his 2013 paper *Walking on real numbers*, which contains striking images of walks in the plane associated with various mathematical constants such as π, e, and Champernowne's number C_4 (formed by concatenating the representations of successive integers in base 4).

In a different context, Jon used visualisation to explain the dynamics of optimisation algorithms such as Douglas-Rachford.

A walk on π

The figure is defined by the first 10^{11} base-4 digits of π. The path moves one unit east, north, west, or south, depending on whether the corresponding digit is $0, 1, 2$, or 3. The colours indicate the overall position in the walk.



A walk on π (base 4)

π is transcendental, but it is not known if it is *4-normal* (i.e. if its base-4 digits are asymptotically uniformly distributed).

Champernowne's number C_4

The figure is defined by the first 10^5 base-4 digits of Champernowne's number C_4. It is known that C_4 is 4-normal.

We might consider using the (base 2) digits of a transcendental number as a pseudo-random number generator. The figure above shows that normality is *not* a sufficient condition for a good random number generator! We would be better off using the digits of π than the digits of C_4 for such a purpose, even though π has not been proved to be 4-normal.

Reproducibility

Jon was concerned with the question of *reproducibility* in computational science (just as we should all be concerned about reproducibility of the results of vaccine trials).

A 2014 paper by Reinhard Ganz in the journal *Experimental Mathematics* claimed that the first 10^{13} decimal digits of π are significantly non-random.

This seemed unlikely to me, so I raised the subject with Jon. As a result, a 2017 paper by Jon (with Bailey, RPB, and Reisi), whose final version was sent to the publisher just a few days before Jon's death, debunks Ganz's claim.

Our paper points out the difficulties in reproducing Ganz's results, and reaches the opposite conclusion, i.e. we conclude that there is no significant statistical evidence for nonrandomness of the (first 10^{13}) decimal digits of π.

History rhymes

"History doesn't repeat itself, but it often rhymes"
– attributed to Mark Twain (1835–1910).

Curiously, an early paper[2] by Metropolis, Reitwiesner and von Neumann studied the first $2,000$ decimal digits of π and e (computed on the ENIAC) and concluded that, according to a χ^2 test, the digits of e were significantly non-random.

This phenomenon disappeared in later computations with more digits.

[2] N. C. Metropolis, G. Reitwiesner and J. von Neumann, Statistical treatment of values of first $2,000$ decimal digits of e and π calculated on the ENIAC, *Math. Tables and Other Aids to Computation* 4 (1950), 109–111. MR 0037598 (12,286j). Also in von Neumann's Collected Works, Vol. 5.

2016–2021

In August 2016, Jon was on leave from Newcastle and on a 4-month visit to Canada as Distinguished Scholar in Residence at Western University, London, Ontario. He died unexpectedly on 2 August 2016 (aged 65). It was a shock to his friends and colleagues, as no one expected it.

Jon was survived by his wife Judith, their three daughters Naomi, Rachel, and Tova, and five grandchildren, as well as by his brother Peter, sister Sarah, and parents David and Bessie.

Sadly, Peter died in August 2020 (aged 67), and David died in September 2021 (aged 97). Our hopes for more "Borwein mathematics" lie with younger generations, although they will probably not have the surname "Borwein".

Jon’s legacy

Jon’s legacy includes his books, papers, and the former students who will carry on his work.

Another of Jon’s legacies is the research centre CARMA that he established on his arrival in Newcastle. Its objectives were “becoming a world-leading institution in: using computers as an adjunct to mathematical discovery; researching and developing computer-based decision-support systems; and promoting use of appropriate tools in academia, education and industry”.

The need for CARMA

It would be a great pity if CARMA died, since there is a need for an Australian organisation with objectives similar to those of CARMA.

The utility of computation in mathematics was forseen by early pioneers such as Turing, Von Neumann, and Shannon, but in their day the technology available was far less powerful than now. Thanks to Moore's law and advances in algorithms and software, that has changed. Computation (both numerical and symbolic) is now indispensable in diverse areas of mathematics as well as in related areas such as statistical mechanics. Still, there is a tendency amongst some pure mathematicians to look down on it, or to fail to appreciate what it can offer.

There is also a tendency for different subject areas to "reinvent the wheel" due to ignorance of what has been done and what software is available in different areas. Hence, an organisation that can bring together researchers from different areas is highly desirable.

Where next for CARMA?

There is no need for CARMA to remain closely linked to the University of Newcastle. Indeed, if Jon were still alive today, he would probably have moved or re-established it elsewhere.

The CARMA Executive is currently considering options for the future of CARMA, not necessarily linked to Newcastle. If you have any ideas, please contact the Director of CARMA (Judy-anne Osborn) or someone else on the Executive.

Meanwhile, CARMA welcomes external members. If you are interested in becoming one and/or participating in CARMA activities, please contact the Director.

More information about CARMA and its history can be found on the website `https://carmamaths.org`.

More Borwein family photos

Let's conclude with some more Borwein family photos.

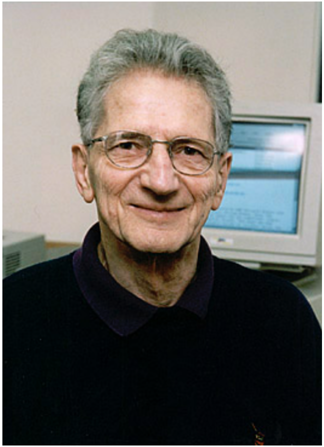
Jon at the Academy of Science, c. 2010.

Atypical, since Jon rarely wore a tie!

Jon in Adelaide, c. 2014.

More typical – no tie and probably wearing shorts

David, aged 90



David Borwein, 2014

The Borwein family, 1999

Peter, David, Jon, Bessie (L to R)
Univ. of Western Ontario, 1999

A few references

Robert Baillie, David Borwein, and Jonathan M. Borwein, Surprising sinc sums and integrals, *Amer. Math. Monthly* 115, 10 (2008), 888–901.

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Judith Borwein, Naomi Borwein, and Brailey Sims, Jonathan M. Borwein, *Gazette AustMS*, Nov. 2017, 289–293.

Richard P. Brent, Jonathan Michael Borwein 1951 – 2016: Life and Legacy, *Maple Transactions* 1, 2 (2021), 13 pp. Also `https://arxiv.org/abs/2107.06030`.
This paper has a much more complete list of references.

Brailey Sims *et al.* (editors), From Analysis to Visualisation: A Celebration of the Life and Legacy of Jonathan M. Borwein, Springer Proc. in Math. and Stat. 313, 2020.