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Automatic Contouring

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ABSTRACT: Automatic Contouring, be it of well-defined functions of two variables or of observational data obtained by experiment, presents certain problems. Various approaches to the solution of some of these problems are discussed and the techniques used in a particular application, namely the generation of contour maps of irregularly spaced gravity data, are presented. A set of conditions to be satisfied by a contouring technique are given.

KEY WORDS AND PHRASES: Maps, contours, least-squares surfaces.
COMPUTING REVIEWS CATEGORIES: 3.14, 5.13.

1. INTRODUCTION

1.1 Background

Early in 1968 the Computer Centre at Monash University was presented with the problem of contouring maps of irregularly spaced data obtained as the result of a helicopter gravity survey. Very little information could be found in the literature relating to contouring procedures (especially for applications involving irregularly spaced data) and hence it was decided to approach the solution to the problem from first principles. As a first step we investigated the problem of drawing contours of functions of two variables. While this gave insight into methods for actually drawing the contours it gave no clue as to how to create the contours from irregularly spaced data.

It is evident that in order to find contours one has first to fit a surface to the data available. Several solutions to this problem are available and have been described in the literature. Maine (1966) discusses in detail the fitting of polynomial surfaces by a method involving several passes over the data, each pass being designed to refine the data in those areas where the initial distribution of points gives rise to an ill-defined surface. Pelto, Elkins and Boyd (1968) describe a weighted least-squares fitting of a polynomial surface and for a certain type of weighting function prove that the surface so fitted is continuous and smooth except possibly near an observation point.

In each of these discussions two points appear to be assumed:

1. It is necessary that the fitted surface pass through each data point.
2. The data is itself free from error.

For a particular application neither of these assumptions may be true and it is evident that if one allows the possibility of error in the data then the first assumption cannot be justified. In the particular application considered, namely a gravity survey, the data cannot be considered to be exactly defined either in location or value.

In this paper we describe how consideration for these errors may be incorporated into a contouring procedure.

1.2 Limitations

If any automatic contouring technique is to be useful then it must satisfy several conditions or limitations.

(a) Any surface fitted to the data should pass through data points whose standard error is insignificant.

(b) Where the standard error of a data point is significantly large a suitable weight should be given to it so that its relative importance in any surface fitting procedure is diminished.

(c) Apparent anomalies must be smoothed out since they may have been introduced by observational error.

(d) Real anomalies must be left in particularly in gravity maps where they are the important features of the contour map.

(e) Compute time should be as small as possible while giving a reasonable accuracy.

(f) If plotting is to be carried out on-line then pen movement should be minimised. (In our case this point is not considered since all plotting is performed off-line.)

(g) The automatic method must produce contours which are acceptable in the sense that they are comparable with hand-drawn contours of the same data. This condition may in fact imply that any fitted surface shall pass through every data point.

It may be argued that this latter consideration is of doubtful validity on the grounds that any automatic method which uses least squares fitting of a surface will necessarily produce the "best" surface and consequently the "best" contours. However, in any weighted process the weights can be chosen in a large number of ways, and since the surface fitted depends largely on the weights the condition is valid and can be used as a means for determining what is a successful weighting scheme.

It is possible that conflicts may arise as a result of conditions (a), (b) and (g). As a general rule, a cartographer uses linear interpolation to obtain contour line positions and consequently hand drawn contours will always fit the data exactly. Hence any user of automatic methods may have to choose between surfaces of best fit in a mathematical sense and surfaces which have been fitted according to some semi-subjective criterion.

2. METHODS

2.1 Surface generation

Since the basic method of weighted least squares has been described in detail (Maine (1966), Pelto et al (1968)) we will discuss in the following only those modifications which have been used to obtain procedures satisfying our (self-imposed) limitations.

In order to satisfy points (a) and (b) we initially chose weights of the form,

$$= W_r \left[k + (1 + d^2/h^2)^{-m} \right]$$

where k, m are constants, d is the distance of the r^{th} data point from the centre of the area over which the surface is to be fitted and W_r a weight associated with the reliability of the data point. While this weighting scheme does give a reasonably acceptable fit for $m = 2$ the resulting contours violate condition (g) in as much as it is impossible to guarantee that the surface will pass through the data points.

To satisfy condition (g) while at the same time giving some thought to errors we investigated a system of weights of the form,

$$= W_r \left| (D - d)/d \right|^m$$

where D is the distance from the centre of the area to the most distant data point considered.

While this does not give complete satisfaction in (a) and (b), unreliable points can be eliminated from consideration by taking $W_r = 0$ and the resulting contours for the case $m = 2$ have been acceptable.

In respect of conditions (c) and (d) the amount of smoothing is related to the order of the surface fitted and also to the number of data points considered for the surface. Fitting of planes has been found to produce reasonable contours even though the smoothing is in many cases accentuated. For this reason, quadratic surface fitting is desirable provided that compute time is kept to a minimun.

Since the major portion of compute time is spent in inversion of matrices obtained from the least squares process this then is the area to be investigated. By economising the inversion process we have been able to reduce the total compute time by about 30 %.

2.2 Contour generation

While there have been detailed accounts of the methods used to develop the surfaces, little has been written on the actual methods of contour drawing and tracking. The method we have developed is simple and straight-forward.

The area to be contoured is first divided into a fairly large number (60 x 60) of sub-areas and within the computer memory a flag is set for each sub-area which determines whether or not a given contour level line passes through the sub-area. Having covered the entire map area in this way a scan process commences, first around the edges of the map area and then into the centre, to determine the starting position of a contour. Once a square is found to have a contour line passing through it a refining process takes place to determine precisely where the contour line cuts an edge. Subsequently, the contour is then tracked by either linear or quadratic interpolation (depending on the type of surface fitting applied) until it either leaves the map area or returns to the starting square. The step size for interpolation along the contour lines will of course determine the maximum curvature of a contour line and hence the smallest feature that can be plotted. It also has a large bearing on the compute time and although we have used a fixed step length it may be that a variable step length method depending on the curvature of the contour line could decrease the overall running time. Preliminary investigations into this would appear to justify this statement.

We should note here that our method of tracking along contours makes no attempt at minimising pen movement. The method in fact closely resembles the direct method described by Maine, Hincksman and Seaman, (1967).

3. CONCLUSION

The results of the contouring methods we have described are dependent in the long run upon certain parameters, namely,

(1) weighting function,

(2) contour interpolation step-length.

By careful selection of these it is possible to obtain a contoured map which satisfies what is in some ways the prime consideration—the map closely resembles a hand-contoured map. We should note however that even though we have a map which is reproducible, the contours are still necessarily subjectively positioned and that, furthermore, it cannot be said that the machine-drawn contours are any "better" or any "worse" than those drawn by hand.

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