

Table of $T_{r,k}$ and $A_{r,k}$ for $r = 1(1)40$, $k = 1(1)r$.

R. Brent, 1 January 1970.

Introduction

The constants c_k , $S_{r,k}$ and $A_{r,k}$ are defined in [1].

We define

$$T_{r,k} = S_{r,k} \cdot \prod_{q \leq r+1} (q - 1),$$

where the product is taken over odd primes $q \geq 3$. It follows from (2.1) and (2.2) of [1] that $T_{r,k}$ is an exact integer for $1 \leq k \leq r$ (also, $T_{r,k}$ must be even unless r is a power of 2).

The constants $T_{r,k}$ were computed exactly for $1 \leq k \leq r \leq 40$, and are given in Table 2. Equation (2.2) of [1] was used, and by considering $q = 3$ separately the number of terms in the sum (2.1) of [1] which need be considered is reduced to $\binom{\lfloor \frac{2}{3}(r-1) \rfloor}{k-1}$. (Here $\lfloor x \rfloor$ means the greatest integer $y \leq x$.) In fact we have, with the notation of [1],

$$T_{r,k} = K_r \cdot \sum_{v \in V, n_i \not\equiv r' \pmod{3}} \prod_{5 \leq q \leq r+1} (q - |L_{k+1}|),$$

where

$$K_r = \begin{cases} 2 & \text{if } r \equiv 0 \pmod{3} \\ 1 & \text{if } r \not\equiv 0 \pmod{3} \end{cases}$$

and

$$r' = \begin{cases} 1 & \text{if } r \equiv 2 \pmod{3} \\ 2 & \text{if } r \not\equiv 2 \pmod{3} \end{cases}.$$

If r is not divisible by 3, a further factor of nearly 2 can be saved by symmetry.

From the $T_{r,k}$ and c_k , the constants $A_{r,k}$ may easily be computed, and the accuracy of the $A_{r,k}$ will depend on the accuracy of the c_k . For completeness, the c_k used in the

computation of Table 2 are given in Table 1.

Computations were performed on a Hewlett-Packard 2116B computer at Stanford University. For $r = 3s, 3s+1$ or $3s+2$, s an integer, the time required to find $T_{r,1}, \dots, T_{r,r}$ was about $(Q + 2) \cdot 2^{2s-26}$ hours, where Q is the number of odd primes less than or equal to $r + 1$.

Reference

1. R. P. Brent, "Empirical Evidence for a Proposed Distribution of Small Prime Gaps", to appear.

Table 1

The constants c_k for $k = 1(1)20$, accurate to 5S (i.e. the error is less than one in the fifth significant digit).

<u>k</u>	<u>c_k</u>
1	0.6601619
2	0.7216032
3	0.4841173
4	0.6508542
5	0.4552960
6	0.7131471
7	0.6291173
8	0.5170453
9	0.3478712
10	0.5331996
11	0.3744102
12	0.6343813
13	0.5626010
14	0.4653414
15	0.3155542
16	0.5057097
17	0.3596735
18	0.6362383
19	0.5779255
20	0.4953785

Table 2

The constants $T_{r,k}$ and $A_{r,k}$ for $r = 1(1)40$, $k = 1(1)r$.
 For each r , entries for which $T_{r,k}$ and $A_{r,k}$ vanish are omitted, except for the first. The $T_{r,k}$ are exact integers, the $A_{r,k}$ should be accurate to 5S (the errors could be 2 in the fifth place for the higher values of k).

<u>r</u>	<u>k</u>	<u>$T_{r,k}$</u>	<u>$A_{r,k}$</u>
1	1	1	.1320324E+01
2	1	1	.1320324E+01
2	2		
3	1	2	.2640647E+01
3	2	2	-.5716497E+01
3	3		
4	1	3	.1320324E+01
4	2	4	-.5716497E+01
4	3	1	.4151183E+01
4	4		
5	1	4	.1760432E+01
5	2	6	-.8574747E+01
5	3	2	.8302366E+01
5	4		
6	1	30	.2640647E+01
6	2	56	-.2000774E+02
6	3	30	.4151184E+02
6	4	4	-.2026362E+02
6	5		
7	1	18	.1584388E+01
7	2	40	-.1429125E+02
7	3	28	.3874438E+02
7	4	6	-.3039543E+02
7	5		
8	1	15	.1320324E+01
8	2	40	-.1429125E+02
8	3	36	.4981421E+02
8	4	12	-.6079086E+02
8	5	1	.1729864E+02
8	6		

9	1	30	.2640647E+01
9	2	92	-.3286987E+02
9	3	100	.1383728E+03
9	4	44	-.2228998E+03
9	5	6	.1037919E+03
9	6		
10	1	180	.1760432E+01
10	2	624	-.2786793E+02
10	3	812	.1605124E+03
10	4	480	-.4052724E+03
10	5	120	.4151677E+03
10	6	8	-.1079442E+03
10	7		
11	1	150	.1467026E+01
11	2	504	-.2250871E+02
11	3	632	.1249309E+03
11	4	350	-.2955111E+03
11	5	72	.2491006E+03
11	6		
12	1	2,970	.2640647E+01
12	2	10,880	-.4859024E+02
12	3	15,642	.3435599E+03
12	4	11,008	-.1161781E+04
12	5	3,780	.1868254E+04
12	6	504	-.1133414E+04
12	7		
13	1	1,620	.1440353E+01
13	2	6,688	-.2986871E+02
13	3	11,090	.2435801E+03
13	4	9,378	-.9897515E+03
13	5	4,224	.2087700E+04
13	6	952	-.2140893E+04
13	7	84	.8318918E+03
13	8		
14	1	1,782	.1584388E+01
14	2	7,400	-.3304852E+02
14	3	12,312	.2704200E+03
14	4	10,400	-.1097613E+04
14	5	4,634	.2290341E+04
14	6	1,008	-.2266828E+04
14	7	80	.7922779E+03
14	8		
15	1	3,960	.3520863E+01
15	2	19,312	-.8624770E+02
15	3	38,958	.8556711E+03
15	4	41,768	-.4408183E+04
15	5	25,376	.1254202E+05
15	6	8,570	-.1927253E+05
15	7	1,446	.1432042E+05
15	8	90	-.3780404E+04
15	9		

16	1	22,275	.1320324E+01
16	2	113,792	-.3629977E+02
16	3	244,829	.4136468E+03
16	4	287,904	-.2532109E+04
16	5	200,805	.9022480E+04
16	6	84,280	-.1895321E+05
16	7	20,583	.2264926E+05
16	8	2,656	-.1394549E+05
16	9	140	.3409503E+04
16	10		
17	1	23,760	.1408345E+01
17	2	122,400	-.3904573E+02
17	3	265,734	.4489665E+03
17	4	315,120	-.2771473E+04
17	5	220,944	.9927357E+04
17	6	92,466	-.2079411E+05
17	7	22,120	.2434055E+05
17	8	2,700	-.1417652E+05
17	9	128	.3117260E+04
17	10		
18	1	757,350	-.2640648E+01
18	2	3,979,008	-.7933173E+02
18	3	8,883,060	.1000549E+04
18	4	10,966,172	-.6889088E+04
18	5	8,160,360	.2820444E+05
18	6	3,743,544	-.7015520E+05
18	7	1,040,798	.1041164E+06
18	8	166,040	-.8718037E+05
18	9	13,644	.3692006E+05
18	10	448	-.6124206E+04
18	11		
19	1	400,950	.1397990E+01
19	2	2,239,104	-.4464228E+02
19	3	5,333,232	.6007118E+03
19	4	7,045,200	-.4425884E+04
19	5	5,612,012	.1939666E+05
19	6	2,737,436	-.5130043E+05
19	7	788,592	.7888694E+05
19	8	120,186	-.6310444E+05
19	9	7,140	.1932053E+05
19	10		
20	1	504,900	.1760432E+01
20	2	2,915,840	-.5813474E+02
20	3	7,236,810	.8151224E+03
20	4	10,062,640	-.6321477E+04
20	5	8,559,382	.2958358E+05
20	6	4,558,512	-.8542798E+05
20	7	1,490,236	.1490760E+06
20	8	279,200	-.1465958E+06
20	9	25,632	.6935906E+05
20	10	768	-.1049864E+05
20	11		

21	1	908,820	.3168777E+01
21	2	5,777,920	-.1151976E+03
21	3	16,006,998	.1802958E+04
21	4	25,293,628	-.1588978E+05
21	5	25,040,302	.8654617E+05
21	6	16,042,408	-.3006399E+06
21	7	6,621,546	.6623875E+06
21	8	1,691,666	-.8882202E+06
21	9	243,872	.6599069E+06
21	10	16,210	-.2215923E+06
21	11	294	.1986281E+05
21	12		

22	1	8,835,750	.1467027E+01
22	2	56,689,920	-.5651295E+02
22	3	159,615,504	.9462307E+03
22	4	258,927,336	-.9036752E+04
22	5	267,006,488	.5428518E+05
22	6	182,041,376	-.2132196E+06
22	7	82,606,080	.5509007E+06
22	8	24,436,076	-.9164517E+06
22	9	4,458,038	.9279426E+06
22	10	445,104	-.5070517E+06
22	11	17,952	.1102589E+06
22	12		

23	1	8,330,850	.1383197E+01
23	2	55,372,800	-.5519994E+02
23	3	161,805,900	.9592158E+03
23	4	272,787,560	-.9520482E+04
23	5	292,530,312	.5947444E+05
23	6	207,276,852	-.2427772E+06
23	7	97,483,328	.6501174E+06
23	8	29,693,850	-.1113639E+07
23	9	5,502,392	.1145325E+07
23	10	541,736	-.6171324E+06
23	11	20,016	.1229357E+06
23	12		

24	1	15,904,350	.2640648E+01
24	2	110,218,240	-.1098742E+03
24	3	337,714,368	.2002035E+04
24	4	601,500,212	-.2099279E+05
24	5	688,462,352	.1399715E+06
24	6	528,267,460	-.6187439E+06
24	7	274,911,048	.1833385E+07
24	8	95,882,496	-.3595981E+07
24	9	21,589,178	.4493797E+07
24	10	2,909,644	-.3314595E+07
24	11	201,728	.1238987E+07
24	12	4,972	-.1620185E+06
24	13		

25	1	10,602,900	.1760432E+01
25	2	78,453,760	-.7820886E+02
25	3	259,208,326	.1536636E+04
25	4	503,845,272	-.1758457E+05
25	5	638,819,850	.1298787E+06
25	6	553,348,174	-.6481202E+06
25	7	333,200,948	.2222120E+07
25	8	139,061,136	-.5215355E+07
25	9	39,323,050	.8185111E+07
25	10	7,178,822	-.8177939E+07
25	11	771,934	.4741120E+07
25	12	40,444	-.1317915E+07
25	13	600	.1019519E+06
25	14		
26	1	8,675,100	.1440353E+01
26	2	65,815,552	-.6561009E+02
26	3	222,480,280	.1318905E+04
26	4	441,115,188	-.1539524E+05
26	5	568,043,690	.1154891E+06
26	6	496,665,190	-.5817290E+06
26	7	299,126,614	.1994878E+07
26	8	123,115,352	-.4617323E+07
26	9	33,552,948	.6984062E+07
26	10	5,667,978	-.6456823E+07
26	11	519,164	.3188638E+07
26	12	18,536	-.6040174E+06
26	13		
27	1	15,904,350	.2640648E+01
27	2	125,480,960	-.1250892E+03
27	3	445,109,722	.2638694E+04
27	4	936,691,472	-.3269120E+05
27	5	1,299,110,200	.2641225E+06
27	6	1,247,084,166	-.1460672E+07
27	7	846,351,674	.5644328E+07
27	8	407,142,718	-.1526950E+08
27	9	136,899,012	.2849559E+08
27	10	31,105,922	-.3543511E+08
27	11	4,478,894	.2750879E+08
27	12	360,574	-.1174973E+08
27	13	11,960	.2032242E+07
27	14		
28	1	257,650,470	.1584389E+01
28	2	2,057,036,800	-.7886983E+02
28	3	7,401,008,550	.1754983E+04
28	4	15,847,906,176	-.2304596E+05
28	5	22,467,234,164	.1986008E+06
28	6	22,193,313,780	-.1181561E+07
28	7	15,655,667,118	.4971798E+07
28	8	7,952,068,360	-.1491172E+08
28	9	2,895,713,696	.3172335E+08
28	10	743,552,820	-.4705756E+08
28	11	130,445,250	.4712810E+08
28	12	14,782,880	-.3010732E+08
28	13	969,120	.1097817E+08
28	14	27,552	-.1711304E+07
28	15		

29	1	222,660,900	.1369225E+01
29	2	1,776,107,520	-.6809859E+02
29	3	6,375,729,282	.1511861E+04
29	4	13,593,395,200	-.1976746E+05
29	5	19,128,593,472	.1690886E+06
29	6	18,668,443,948	-.9938989E+06
29	7	12,918,700,344	.4102614E+07
29	8	6,366,803,478	-.1193903E+08
29	9	2,211,452,320	.2422708E+08
29	10	527,416,896	-.3337888E+08
29	11	82,484,676	.2980059E+08
29	12	7,844,956	-.1597731E+08
29	13	399,104	.4521043E+07
29	14	8,100	-.5031055E+06
29	15		

30	1	16,604,141,400	.3520864E+01
30	2	137,449,159,680	-.1882144E+03
30	3	514,590,231,096	.4519390E+04
30	4	1,151,172,876,672	-.6438580E+05
30	5	1,712,427,905,900	.6054854E+06
30	6	1,783,275,849,600	-.3955866E+07
30	7	1,332,662,130,802	.1840070E+08
30	8	720,521,099,012	-.6141461E+08
30	9	280,433,370,000	.1462965E+09
30	10	77,181,262,520	-.2442304E+09
30	11	14,532,377,696	.2763341E+09
30	12	1,774,107,900	-.2007338E+09
30	13	128,975,226	.8594286E+08
30	14	4,843,936	-.1880409E+08
30	15	67,200	.1502464E+07
30	16		

31	1	6,441,261,750	.1365853E+01
31	2	56,911,196,160	-.7793071E+02
31	3	228,830,896,980	.2009708E+04
31	4	553,838,648,256	-.3097653E+05
31	5	899,233,690,940	.3179537E+06
31	6	1,033,124,953,150	-.2291795E+07
31	7	863,141,800,752	.1191780E+08
31	8	530,478,526,848	-.4521606E+08
31	9	239,751,903,028	.1250738E+09
31	10	78,777,761,916	-.2492824E+09
31	11	18,369,533,720	.3492978E+09
31	12	2,915,154,534	-.3298390E+09
31	13	293,137,524	.1953327E+09
31	14	16,351,382	-.6347583E+08
31	15	367,552	.8217762E+07
31	16		

32	1	6,226,553,025	.1320324E+01
32	2	54,069,370,880	-.7403926E+02
32	3	213,249,174,660	.1872862E+04
32	4	505,037,201,552	-.2824703E+05
32	5	799,957,727,050	.2828514E+06
32	6	893,157,886,368	-.1981304E+07
32	7	721,531,916,562	.9962530E+07
32	8	425,908,066,056	-.3630286E+08
32	9	183,169,043,337	.9555561E+08
32	10	56,517,842,560	-.1788436E+09
32	11	12,135,950,902	.2307659E+09
32	12	1,721,143,800	-.1947411E+09
32	13	147,584,208	.9834297E+08
32	14	6,550,848	-.2543030E+08
32	15	112,320	.2511261E+07
32	16		
33	1	13,836,784,500	.2934054E+01
33	2	127,085,826,560	-.1740236E+03
33	3	532,897,145,178	.4680170E+04
33	4	1,350,034,178,880	-.7550823E+05
33	5	2,304,270,553,388	.8147507E+06
33	6	2,797,174,955,788	-.6205012E+07
33	7	2,484,362,101,368	.3430275E+08
33	8	1,635,525,293,974	-.1394062E+09
33	9	799,505,921,412	.4170862E+09
33	10	287,863,227,268	-.9109069E+09
33	11	74,936,047,534	.1424914E+10
33	12	13,668,343,964	-.1546523E+10
33	13	1,663,472,704	.1108458E+10
33	14	125,284,200	-.4863515E+09
33	15	5,175,900	.1157233E+09
33	16	91,350	-.1162170E+08
33	17		
34	1	6,641,656,560	.1408346E+01
34	2	63,757,824,000	-.8730606E+02
34	3	280,475,561,070	.2463277E+04
34	4	748,667,274,256	-.4187343E+05
34	5	1,353,200,542,126	.4784687E+06
34	6	1,749,907,156,172	-.3881842E+07
34	7	1,667,427,651,782	.2302296E+08
34	8	1,187,717,946,838	-.1012368E+09
34	9	634,711,470,646	.3311163E+09
34	10	253,000,600,276	-.8005887E+09
34	11	74,055,977,086	.1408179E+10
34	12	15,480,295,878	-.1751538E+10
34	13	2,207,599,238	.1471038E+10
34	14	199,031,808	-.7726386E+09
34	15	9,862,880	.2205152E+09
34	16	192,160	-.2444693E+08
34	17		

35	1	9,962,484,840	.2112519E+01
35	2	96,876,953,600	-.1326574E+03
35	3	432,135,635,850	.3795231E+04
35	4	1,171,099,054,240	-.6550030E+05
35	5	2,152,382,571,534	.7610457E+06
35	6	2,835,785,493,016	-.6290663E+07
35	7	2,759,909,244,350	.3810737E+08
35	8	2,014,530,552,238	-.1717113E+09
35	9	1,108,039,672,762	.5780421E+09
35	10	457,350,869,200	-.1447230E+10
35	11	139,837,955,468	.2659028E+10
35	12	30,944,771,326	-.3501287E+10
35	13	4,778,251,840	.3183997E+10
35	14	487,520,528	-.1892548E+10
35	15	30,442,852	.6806442E+09
35	16	1,064,164	-.1353848E+09
35	17	18,816	.1344756E+08
35	18		
36	1	435,858,711,750	.2640647E+01
36	2	4,341,200,912,384	-.1748404E+03
36	3	19,890,493,216,704	.5293585E+04
36	4	55,548,398,353,920	-.9708920E+05
36	5	105,609,934,037,258	.1204576E+07
36	6	144,585,042,777,240	-.1069116E+08
36	7	147,013,719,890,738	.6999606E+08
36	8	112,853,773,847,024	-.3435442E+09
36	9	65,818,149,695,190	.1271703E+10
36	10	29,109,653,091,480	-.3542836E+10
36	11	9,668,529,195,900	.7353905E+10
36	12	2,367,339,177,312	-.1116064E+11
36	13	414,768,995,340	.1201659E+11
36	14	49,670,750,140	-.8764586E+10
36	15	3,784,555,236	.4029301E+10
36	16	162,368,120	-.1032837E+10
36	17	2,979,504	.1120742E+09
36	18		
37	1	224,155,908,900	.1358048E+01
37	2	2,258,481,971,200	-.9095956E+02
37	3	10,485,155,702,790	.2790481E+04
37	4	29,727,848,617,632	-.5195923E+05
37	5	57,509,346,598,784	.6559457E+06
37	6	80,322,576,915,068	-.5939351E+07
37	7	83,574,029,162,340	.3979120E+08
37	8	65,878,208,525,724	-.2005434E+09
37	9	39,608,684,408,832	.7652979E+09
37	10	18,136,932,095,412	-.2207385E+10
37	11	6,264,191,334,804	.4764557E+10
37	12	1,600,844,158,000	-.7547059E+10
37	13	293,085,317,616	.8491202E+10
37	14	36,456,896,856	-.6432953E+10
37	15	2,811,462,016	.2993277E+10
37	16	112,121,310	-.7132125E+09
37	17	1,405,120	.5285366E+08
37	18		

38	1	230,748,729,750	.1397990E+01
38	2	2,411,472,967,680	-.9712120E+02
38	3	11,646,227,156,256	.3099485E+04
38	4	34,464,227,113,728	-.6023764E+05
38	5	69,858,296,785,638	.7967964E+06
38	6	102,699,532,806,420	-.7593987E+07
38	7	113,090,240,638,294	.5384445E+08
38	8	94,980,077,296,328	-.2891339E+09
38	9	61,362,749,906,514	.1185618E+10
38	10	30,530,722,672,060	-.3715790E+10
38	11	11,633,891,012,650	.8848763E+10
38	12	3,353,415,754,800	-.1580943E+11
38	13	716,559,419,004	.2075999E+11
38	14	110,184,341,212	-.1944243E+11
38	15	11,696,423,466	.1245283E+11
38	16	810,181,320	-.5153627E+10
38	17	33,751,448	.1269563E+10
38	18	689,472	-.1517005E+09
38	19		
39	1	475,482,231,000	.2880706E+01
39	2	5,056,905,641,984	-.2036651E+03
39	3	24,879,391,641,270	.6621311E+04
39	4	75,091,338,143,520	-.1312470E+06
39	5	155,456,237,650,394	.1773118E+07
39	6	233,797,081,544,148	-.1728782E+08
39	7	263,891,934,090,706	.1256440E+09
39	8	227,713,534,939,218	-.6931952E+09
39	9	151,586,934,803,914	.2928881E+10
39	10	77,985,040,623,608	-.9491290E+10
39	11	30,856,603,351,642	.2346960E+11
39	12	9,281,105,407,498	-.4375508E+11
39	13	2,080,266,780,954	.6026903E+11
39	14	336,823,629,344	-.5943378E+11
39	15	37,538,402,896	.3996599E+11
39	16	2,661,185,116	-.1692801E+11
39	17	103,715,324	.3901258E+10
39	18	1,553,664	-.3418436E+09
39	19		
40	1	11,332,326,505,500	.1760432E+01
40	2	122,267,833,425,920	-.1295867E+03
40	3	611,333,513,121,420	.4397246E+04
40	4	1,878,969,903,287,040	-.9122559E+05
40	5	3,970,507,565,362,870	.1293920E+07
40	6	6,111,795,030,108,760	-.1329204E+08
40	7	7,083,406,761,250,116	.1021984E+09
40	8	6,300,241,614,515,392	-.5993412E+09
40	9	4,343,194,458,469,710	.2706997E+10
40	10	2,327,366,695,151,460	-.9441859E+10
40	11	966,415,973,394,924	.2534683E+11
40	12	308,155,698,303,488	-.5188490E+11
40	13	74,292,815,006,406	.7971840E+11
40	14	13,234,573,074,528	-.8981889E+11
40	15	1,687,552,260,700	.7186746E+11
40	16	147,779,920,800	-.3916834E+11
40	17	8,452,964,462	.1382430E+11
40	18	295,737,904	-.2957705E+10
40	19	5,189,184	.3143934E+09
40	20		

41	1	8,717,174,235,000	.1354178E+01
41	2	95,681,838,120,960	-.1014093E+03
41	3	487,113,052,565,106	.3503744E+04
41	4	1,525,850,855,944,192	-.7408137E+05
41	5	3,289,438,770,811,056	.1071972E+07
41	6	5,171,436,235,308,420	-.1124693E+08
41	7	6,128,844,426,141,488	.8842616E+08