THE COMPLEXITY OF MULTIPLE-PRECISION ARITHMETIC

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Abstract

In studying the complexity of iterative processes it is usually assumed that the arithmetic operations of addition, multiplication, and division can be performed in certain constant times. This assumption is invalid if the precision required increases as the computation proceeds. We give upper and lower bounds on the number of single-precision operations required to perform various multiple-precision operations, and deduce some interesting consequences concerning the relative efficiencies of methods for solving nonlinear equations using variable-length multiple-precision arithmetic.

Comments

Only the Abstract is given here. The full paper appeared as [3]. Related papers (written later) are [2, 4].

Errata

Page 141, line 3: replace "Proof" by "Remark".
Page 157, last line: replace "<" by ">".

References

[1] R. S. Anderssen and R. P. Brent (editors), *The Complexity of Computational Problem Solving*, University of Queensland Press, Brisbane, 1976, 262 pp. LC 76-374278, ISBN 0-7022-1213-X. rpb031.

[2] R. P. Brent, "Multiple-precision zero-finding methods and the complexity of elementary function evaluation", in *Analytic Computational Complexity* (edited by J. F. Traub), Academic Press, New York, 1975, 151–176. MR 52#15938, 54#11843; Zbl 342.65031. rpb028.

[3] R. P. Brent, "The complexity of multiple-precision arithmetic", in [1, 126–165]. Retyped in LaTeX, Oxford, 1999. rpb032.

[4] R. P. Brent, "Fast multiple-precision evaluation of elementary functions", *J. ACM* 23 (1976), 242–251. MR 52#16111, Zbl 324.65018. rpb034.

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