FAST ALGORITHMS FOR COMPOSITION AND REVERSION OF MULTIVARIATE POWER SERIES

(Extended Abstract)

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Our earlier paper, "Fast Algorithms for Manipulating Formal Power Series," dealt with power series in one variable. In this paper, we give fast algorithms for manipulating <u>multivariate</u> power series. Some of our results on the composition problem for bivariate power series are stated in the following.

We deal with power series over an algebraically complete field, and count operations defined by the field. Let $Q(s,t) = \sum_{i,j=0}^{\infty} q_{i,j} s^i t^j$ be a bivariate power series. We define the degree of the term $q_{i,j}\, s^i t^j$ to be $i+j$. By computing $Q(s,t) \bmod (s+t)^{n+1}$, we mean computing the $q_{i,j}$ for all i,j such that $i + j \le n$.

<u>Theorem 1</u>

Given bivariate power series Q, and univariate series P_1, P_2 with no constant terms, $R(s) = Q(P_1(s),P_2(s)) \bmod s^{n+1}$ can be computed in $O(n^2 \log n)$ operations.

<u>Theorem 2</u>

Given univariate power series Q and bivariate series P with no constant term, $R(s,t) = Q(P(s,t)) \bmod (s+t)^{n+1}$ can be computed in $O(n^{2.5} \log n)$ operations.

<u>Theorem 3</u>

Given bivariate power series P_1, P_2 and Q, where P_1 and P_2 have no constant terms, $R(s,t) = Q(P_1(s,t),P_2(s,t)) \bmod (s+t)^{n+1}$ can be computed in $O(n^3 \log n)$ operations.

Note that for $k = 1,2,3$ the classical bound for the composition problem considered in Theorem k above is $O(n^{k+3})$. We have now reduced the bound to $O(n^{\frac{1}{2}(k+3)} \log n)$, for $k = 1,2,3$.

The close relationship between the composition and reversion problems for the one variable case has been pointed out in our earlier paper. For the multivariate case, this relationship becomes even more important. We have been able to derive fast reversion algorithms through fast composition algorithms, and vice versa. For example, the algorithm which establishes Theorem 3 uses a bivariate reversion algorithm to perform the composition. On the other hand, by the results of Theorems 1, 2 and 3 about composition, we have obtained fast algorithms for the bivariate reversion problem through the use of Newton-like iterations.

Results on power series with more than two variables are not stated in this abstract, but they will be included in the paper.