

COMPUTATION OF THE REGULAR CONTINUED FRACTION FOR EULER'S CONSTANT

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ABSTRACT

We describe a computation of the first 20,000 partial quotients in the regular continued fraction for Euler's constant $\gamma = 0.577\dots$ and $\exp(\gamma) = 1.781\dots$. A preliminary step was the calculation of γ and $\exp(\gamma)$ to 20,700D. It follows from the continued fractions that, if γ or $\exp(\gamma)$ is of the form P/Q for integers P and Q , then $|Q| > 10^{10000}$.

COMMENTS

Only the Abstract is given here. The full paper appeared as [2]. The result improved on an earlier (and only partially correct) result of Beyer and Waterman [1]. An interesting connection with the work of Ramanujan is described in [4]. For a sequel which introduced some more efficient algorithms and extended the computation, see [3].

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- [4] R. P. Brent, *An asymptotic expansion inspired by Ramanujan*, Technical Report CMA-MR02-93/SMS-10-93, Centre for Mathematics and its Applications, ANU, February 1993, 7 pp. Also "Ramanujan and Euler's constant", to be presented at *Mathematics of Computation 1943–1993*, Vancouver, August 1993. rpb139.

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Daniel Shanks made several useful suggestions and drew my attention to the disagreement with Beyer and Waterman's computation.

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