NUMERICALLY STABLE SOLUTION OF DENSE SYSTEMS OF LINEAR EQUATIONS USING MESH-CONNECTED PROCESSORS

A. BOJANCZYK, R. P. BRENT, AND H. T. KUNG

Abstract

We propose a multiprocessor structure for solving a dense system of n linear equations. The solution is obtained in two stages. First, the matrix of coefficients is reduced to upper triangular form via Givens rotations. Second, a back substitution process is applied to the triangular system. A two-dimensional array of $\Theta(n^2)$ processors is employed to implement the first stage, and (using a previously known scheme) a one-dimensional array of $\Theta(n)$ processors is employed to implement the second stage. These processor arrays allow both stages to be carried out in time $O(n)$, and they are well suited for VLSI implementation as identical processors with a simple and regular interconnection pattern are required.

Comments

Only the Abstract is given here. The full paper appeared as [1]. Although publication of [1] was delayed, it predated the well-known work of Gentleman and Kung [3]. As shown in [2], the algorithms of [1] and [3] are in a certain sense dual. The algorithm of [3] is preferable for QR factorization of $m \times n$ matrices where $m > n$ (as in least squares problems) because the size of the processor array depends on n but not on m. For later work, see [2, 4].

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(Bojanczyk) Institute for Informatics, University of Warsaw, PKiN, p. 850, 00-901 Warsaw, Poland

(Brent) Department of Computer Science, Australian National University, Canberra, Australia

(Kung) Department of Computer Science, Carnegie-Mellon University, Pittsburgh, PA 15213, USA