

# SOME INTEGER FACTORIZATION ALGORITHMS USING ELLIPTIC CURVES

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## ABSTRACT

Lenstra's integer factorization algorithm is asymptotically one of the fastest known algorithms, and is ideally suited for parallel computation. We suggest a way in which the algorithm can be speeded up by the addition of a second phase. Under some plausible assumptions, the speedup is of order  $\ln(p)$ , where  $p$  is the factor which is found. In practice the speedup is significant. We mention some refinements which give greater speedup, an alternative way of implementing a second phase, and the connection with Pollard's " $p - 1$ " factorization algorithm.

## COMMENTS

Only the Abstract is given here. The full report appeared as [1]. A revision appeared as [2].

## ERRATA

1. Equations (7.3) and (7.7) have obvious (easily corrected) errors.
2. In equation (1.1), the " $(1 + o(1))$ " factor should be *inside* the exponential.

## REFERENCES

- [1] R. P. Brent, "Some integer factorization algorithms using elliptic curves", Report CMA-R32-85, Centre for Mathematical Analysis, ANU, September 1985, 20 pp. rpb097.
- [2] R. P. Brent, "Some integer factorization algorithms using elliptic curves", *Proc. Ninth Australian Computer Science Conference*, special issue of *Australian Computer Science Communications* 8 (1986), 149–163. rpb102.

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