IMPROVED TECHNIQUES FOR LOWER BOUNDS FOR ODD PERFECT NUMBERS

RICHARD P. BRENT, GRAEME L. COHEN, AND H. J. J. TE RIELE

Abstract

If N is an odd perfect number, and $q^k \| N$, q prime, k even, then it is almost immediate that $N > q^{2k}$. We prove here that, subject to certain conditions verifiable in polynomial time, in fact $N > q^{5k/2}$. Using this and related results, we are able to extend the computations in an earlier paper [1] to show that $N > 10^{300}$.

Comments

Only the Abstract is given here. The full paper appeared as [2]. The main part of the proof that there is no odd perfect number N less than 10^{300} is a (very large) tree, each of whose 12655 leaves gives either a contradiction or a sufficiently large lower bound on N. The proof tree is available by anonymous ftp in a separate file `rpb116p.dvi.Z` . The integer factorizations used in the proof are also available by anonymous ftp: see [3].

References

[1] R. P. Brent, and G. L. Cohen, "A new lower bound for odd perfect numbers", *Mathematics of Computation* 53 (1989), 431–437. Supplement, *ibid*, S7–S24. MR 89m:11008. Also appeared as Report TR-CS-88-05, Computer Sciences Laboratory, ANU, February 1988, 50 pp. rpb100.

[2] R. P. Brent, G. L. Cohen and H. J. J. te Riele, "Improved techniques for lower bounds for odd perfect numbers", *Mathematics of Computation* 57 (1991), 857–868. MR 92c:11004. Also appeared as Report NM-R8921, Centrum voor Wiskunde en Informatica, Amsterdam, October 1989, 13 pp. A longer version was submitted to the *Mathematics of Computation* UMT file and appeared as Report CMA-R50-89, CMA, ANU, October 1989, 198 pp. rpb116.

[3] R. P. Brent, *Factor: An Integer Factorization Program for the IBM PC*, Report TR-CS-89-23, Computer Sciences Laboratory, ANU, October 1989, 7 pp. rpb117.

(Brent) Computer Sciences Laboratory, Australian National University, Canberra
E-mail address: `rpb@cslab.anu.edu.au`

(Cohen) School of Mathematical Sciences, University of Technology, Sydney, PO Box 123, Broadway, NSW 2007, Australia

(te Riele) Centrum voor Wiskunde en Informatica, Kruislaan 413, 1098 SJ Amsterdam, The Netherlands
E-mail address: `herman@cwi.nl`

rpb116a typeset using $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-LaTeX.