

# Minimum-Energy All-to-All Multicasting in Wireless Ad Hoc Networks

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**Abstract**—A wireless ad hoc network consists of mobile nodes that are powered by batteries. The limited battery lifetime imposes a severe constraint on the network performance, energy conservation in such a network thus is of paramount importance, and energy efficient operations are critical to prolong the lifetime of the network. All-to-all multicasting is one fundamental operation in wireless ad hoc networks, in this paper we focus on the design of energy efficient routing algorithms for this operation. Specifically, we consider the following minimum-energy all-to-all multicasting problem.

Given an all-to-all multicast session consisting of a set of terminal nodes in a wireless ad hoc network, where the transmission power of each node is either fixed or adjustable, assume that each terminal node has a message to share with each other, the problem is to build a shared multicast tree spanning all terminal nodes such that the total energy consumption of realizing the all-to-all multicast session by the tree is minimized. We first show that this problem is NP-Complete. We then devise approximation algorithms with guaranteed approximation ratios. We also provide a distributed implementation of the proposed algorithm. We finally conduct experiments by simulations to evaluate the performance of the proposed algorithm. The experimental results demonstrate that the proposed algorithm significantly outperforms all the other known algorithms.

**Index Terms**—Wireless ad hoc networks, approximation algorithm, routing algorithms, energy conservation, energy consumption optimization, all-to-all multicasting.

## I. INTRODUCTION

IN recent years, multi-hop wireless ad hoc networks have been receiving significant attention due to their potential applications from civil to military domains. A wireless ad hoc network consists of mobile nodes that are powered by batteries. The limited battery lifetime imposes a severe constraint on the network performance. Energy conservation in such a network thus is of paramount importance, and energy efficient operations are critical to prolong the lifetime of the network. Multicasting of one-to-many or many-to-many communication is one of fundamental operations in any modern telecommunication network including the wireless ad hoc network. To prolong the network lifetime, it is highly

desirable to develop energy efficient routing algorithms for multicasting to optimize its energy consumption.

### A. Related Work

Much effort on multicasting so far has been focused on one-to-many communication pattern. Extensive studies on devising energy-efficient routing algorithms for this type of multicasting have been conducted in past years, and energy efficient algorithms for finding Minimum-Energy Broadcast Tree (MEBT) and Minimum-Energy Multicast Tree (MEMT) have been proposed [2], [3], [10], [11], [13], [14]. However, little attention has ever been paid to *all-to-all multicasting* that follows many-to-many communication pattern in the literature. The applications of all-to-all multicasting can be found in many scenarios such as ad hoc classrooms, convention center, distributed games, teleconferencing, etc. As all-to-all multicasting is one of fundamental operations in wireless ad hoc networks, it is crucial to realize this operation efficiently by minimizing its energy consumption in order to prolong the network lifetime. We refer to this problem as *the minimum-energy all-to-all multicasting problem*.

Ideally, the minimum-energy all-to-all multicasting problem can be solved by employing one-to-many communication mechanism. That is, an energy efficient multicast tree rooted at each terminal node and spanning the other terminal nodes is found first, using any of the proposed algorithms [2], [3], [10], [11], [13], [14]. Each terminal node then uses the exclusive multicast tree rooted at itself to multicast its message to the other terminal nodes. As a result, the total energy consumption of realizing an all-to-all multicast session is the sum of energy consumptions of these multicast trees, which is the minimum one. In real world, this approach, however, may never work, because it requires that each node has large storage to store its neighboring information in each tree and different multicast trees have different sets of participating nodes. When shifting from the current multicast tree to the next multicast tree, the next multicast tree needs to be reconfigured by changing participating nodes and setting the transmission power of each chosen node. The delay and the energy overhead incurred on frequent shifting from one multicast tree to another multicast tree are not negligible, which can be illustrated by the following example: Consider that a group of passengers in an airport terminal waiting for a flight are interested in playing a game together. Assume that each passenger in the group has a wireless access device like a laptop or PDA available and these wireless access devices form a temporary wireless ad hoc network. Assume that each time only one player in the group can play and

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the others will be informed the playing result by the player after he finished his turn. If each player uses an exclusive multicast tree rooted at the player to multicast his playing result to others, then it will result in an unexpected delay for the next player, due to the shifting delay from the current multicast tree to the next multicast tree rooted at the next player. Also, the energy overhead on the reconfiguration of the next multicast tree is not negligible. A more realistic solution to this problem is to build just one multicast tree shared by all the players. Once a player finishes his turn, he can send his playing results to the other players, using the shared multicast tree. The next player then can play on no time. In comparison with the ideal approach, this latter approach incurs neither any energy overhead on the tree reconfiguration nor any shift delay. Furthermore, this approach makes the shared multicast tree maintenance much simpler, because the transmission power of each node in the tree has been fixed, and there is no need to adjust it to suit for different multicast trees. It should be mentioned that this *shared multicast tree* based approach is not new, which has been proposed for wireless ad hoc networks for a while [5], [7], [15]. For example, Wu et al [15] proposed an on-demand protocol called *Ad Hoc Multicast Routing Protocol Utilizing Increasing ID Numbers (AMRIS)*, which constructs a shared multicast tree to support multiple senders and receivers in a multicast session. AMRIS dynamically assigns an ID number to each node in a multicast session. Based on the ID number, a multicast tree rooted at a special node with Smallest-ID (Sid) is created, and the ID number increases as the tree expands from the Sid. Generally speaking, Sid is the source or the node that initiates a multicast session. Chiang et al [5] proposed an adaptive, shared multicast tree that combines shared tree and source tree benefits to realize all-to-all multicasting, under the node mobility environment. Ji and Corson [7] presented a lightweight, adaptive multicast routing protocol that is built upon the Temporally-Ordered Routing Algorithm (TORA) [12], which conceptually is an integration of the CORE Based Tree (CBT) multicast routing protocol [1] and TORA. Nevertheless, all these algorithms for finding shared multicast trees are heuristic algorithms, and none of them explicitly incorporated the energy consumption as an optimization metric into their problem formalization. In contrast, we investigate all-to-all multicasting by devising very first approximation algorithms with guaranteed approximation ratios for it, after taking into account the energy optimization metric on the problem formulation.

## B. Contributions

In this paper we study the minimum-energy all-to-all multicasting problem of terminal set  $D$  in a wireless ad hoc network with the transmission power being either fixed or adjustable. We first show that the problem is NP-Complete. We then devise approximation algorithms with approximation ratio of either  $2(k+1)$  or  $8l/l_{min}$ , depending on whether the transmission power of each node is fixed, where  $k = |D|$  is the number of terminal nodes,  $l_v$  is the message length at terminal node  $v \in D$ ,  $l = \sum_{v \in D} l_v$ , and  $l_{min} = \min_{v \in D} \{l_v\}$ . We also provide a distributed implementation of the proposed algorithm. The distributed algorithm takes  $O(kn)$  time and

requires  $O(km)$  messages, and the solution delivered is within  $4kl/l_{min}$  times of the optimum, where  $n$  and  $m$  are the number of nodes and links in the network. We finally conduct experiments by simulations to evaluate the performance of the proposed algorithm against existing algorithms. The experimental results demonstrate that the proposed approximation algorithm outperforms the other algorithms significantly.

The rest of the paper is organized as follows. Section II introduces the wireless communication model and the problem definition. Section III shows the minimum-energy all-to-all multicasting problem is NP-Complete. Sections IV and V propose approximation algorithms for the problem when the transmission power of each node is either fixed or adjustable. Section VI provides a distributed implementation of the proposed algorithm. Section VII conducts experimental simulation to evaluate the performance of the proposed algorithm. The conclusions are given in Section VIII.

## II. PRELIMINARIES

### A. Wireless communication model

A wireless ad hoc network can be modeled by an undirected graph  $M = (N, A)$ , where  $N$  is the set of homogeneous stationary nodes and  $A$  is the set of links with  $n = |N|$  and  $m = |A|$ . There is a link  $(u, v) \in A$  if nodes  $u$  and  $v$  are within the transmission range of each other, and  $u$  and  $v$  are *neighboring nodes*. Although the network topology is allowed to change due to node mobility, we assume that it is stable during the period from the system response to an all-to-all multicast request to the realization or rejection of the request. We also assume that each node is equipped with omnidirectional antenna and powered by energy-limited batteries. We adopt two transmission models: One is that each node has only one fixed, identical transmission power  $t_e$ . Another is that each node can adjust its transmission power dynamically. We assume that the reception power consumption at each node is  $r_e$ . Clearly  $r_e < t_e$ . Given two nodes  $u$  and  $v$  separated by a distance  $d_{u,v}$ , to guarantee that they are within the transmission range of each other, the minimum transmission power needed at either of them is modeled to be proportional to  $d_{u,v}^\alpha$ , assuming that the proportionality constant is 1 for notational simplicity,  $\alpha$  is a parameter that typically takes a value between 2 and 4, depending on the characteristics of the communication medium. In this paper we assume  $\alpha = 2$ .

### B. The minimum-energy all-to-all multicasting problem

Given a wireless ad hoc network  $M(N, A)$  and an all-to-all multicast session with a terminal set  $D$ , assume that each terminal node  $v \in D \subseteq N$  has a message of length  $l_v$  to share with the others in  $D$ , the *minimum-energy all-to-all multicasting problem* is to construct a shared multicast tree spanning the nodes in  $D$  such that the total energy consumption of realizing the all-to-all multicast session using the tree is minimized. Typically, a mobile node consumes energy in data transmission, computing and reception. For the sake of simplicity, we here just consider the transmission and reception energy consumptions of a node by ignoring its other energy consumptions, since it is well known that the wireless communication is the dominant energy consumption

TABLE I  
NOTATIONS

Notation	Description
$M(N, A)$	The ad hoc network with node set $N$ and link set $A$
$G(V, E, \gamma)$	Communication graph from $M(N, A)$ , $V = N$ , $E = A$ , and $\gamma \mapsto R^+$
$n$	Number of nodes in $M(N, A)$ , $n =  N $
$m$	Number of links in $M(N, A)$ , $m =  A $
$t_e$	Fixed transmission power at each node in $M(N, A)$
$r_e$	Reception power at each node in $M(N, A)$
$d_{u,v}$	Distance between nodes $u$ and $v$
$D$	Terminal set, $D \subseteq N$
$k$	Number of terminal nodes, $k =  D $
$l_v$	Length of the message originated at node $v \in D$
$l_{min}$	$l_{min} = \min_{v \in D} \{l_v\}$
$l_{max}$	$l_{max} = \max_{v \in D} \{l_v\}$
$l$	$l = \sum_{v \in D} l_v$
$T$	A multicast tree in $M$ spanning the nodes in $D$
$N_T$	Set of nodes in tree $T$
$EG(T)$	Set of links (or edges) in tree $T$
$n_T$	Number of nodes in tree $T$ , $n_T =  N_T $
$n_T^1$	Number of nodes in $T$ with degree one, $n_T^1 =  N_T^1 $
$\mathcal{E}_T$	Total amount of energy for realizing an all-to-all multicast session, using $T$
$w_T(v)$	The amount of transmission power of node $v$ in $T$
$T_{opt}^B$	Optimal broadcast tree for an all-to-all broadcast session in $M(N, A)$
$n_1$	Number of leaf nodes in $T_{opt}^B$
$T_D$	The minimum energy transmission multicast tree in $M(N, A)$
$T_D^r$	The corresponding multicast tree in $G(V, E, \gamma)$ of $T_D$ in $M(N, A)$
$p(v)$	Parent of node $v$ in $T_D^r$
$C(v)$	Child set of node $v$ in $T_D^r$
$T_{opt}$	Optimal multicast tree for an all-to-all multicast session in $M(N, A)$
$T_{opt}^{edge}$	A minimum edge-weighted Steiner tree in $G(V, E, \gamma)$
$T_{app}$	An approximate, minimum edge-weighted Steiner tree in $G(V, E, \gamma)$
$N_{app}(v)$	Set of Neighboring nodes in tree $T_{app}$
$T_{opt}''$	The corresponding multicast tree in $M$ of $T_{opt}^{edge}$ in $G$
$T_v$	A shortest path tree in $G(V, E, \gamma)$ rooted at $v$
$E_{in}(T)$	Sum of transmission power of internal nodes in $T$
$E_{out}(T)$	Sum of transmission power of degree-one nodes in $T$
$E(T)$	Total amount of transmission power of the nodes in $T$

in wireless networks. When  $D = N$ , the problem is referred to as the *minimum-energy all-to-all broadcasting problem*. For the sake of convenience, all symbols in the paper are listed in Table I.

### III. NP-COMPLETENESS

In this section we show the minimum-energy all-to-all broadcasting problem is NP-Complete, by a reduction from the *maximum leaf spanning tree problem* (MLST for short) that is to find a spanning tree in an undirected graph  $G(V, E)$  such that the number of leaf nodes in the tree is maximized, while this latter one has been shown to be NP-Complete (ND2) [6].

*Theorem 1:* The minimum-energy all-to-all broadcasting problem in  $M(N, A)$  is NP-Complete.

*Proof:* We reduce MLST to the minimum-energy all-to-all broadcasting problem as follows. Given an instance  $G(V, E)$  of MLST, there is an instance of a wireless ad hoc network  $M(N, A)$  for the minimum-energy all-to-all broadcasting problem, where  $N$  is the set of nodes in  $V$ . Each node in  $N$  has a fixed transmission power  $t_e$  and zero reception power ( $r_e = 0$ ).  $(u, v) \in A$  if there is an edge  $(u, v) \in E$ . Consider an all-to-all broadcast session where each node has a unit-length message to share with each other. Let  $T_{opt}^B$  be the optimal broadcast tree in  $M$  for the all-to-all broadcast session and  $n_1$  the number of leaf nodes in  $T_{opt}^B$ . Then, the

total energy consumption of realizing this session by  $T_{opt}^B$  is  $\mathcal{E}_{T_{opt}^B} = (n - n_1) * (n - n_1) * t_e + n_1 * (n - n_1) * t_e + n_1 * t_e = n^2 * t_e - (n - 1) * n_1 * t_e$ , where term  $(n - n_1) * (n - n_1) * t_e$  and term  $n_1 * (n - n_1) * t_e + n_1 * t_e$  are the total energy consumptions of internal nodes and leaf nodes in  $T_{opt}^B$  for broadcasting their messages. Clearly,  $\mathcal{E}_{T_{opt}^B}$  is minimized when  $n_1$  is maximized, given both  $n$  and  $t_e$ .

MLST thus can be reduced to the minimum-energy all-to-all broadcasting problem in polynomial time. To show the problem of concern is in NP is straightforward, it is omitted. Thus, the problem is NP-Complete. ■

*Corollary 1:* The minimum-energy all-to-all multicasting problem in  $M(N, A)$  is NP-Complete.

### IV. APPROXIMATION ALGORITHM FOR FIXED TRANSMISSION POWER

In this section we assume that the transmission power of each node is fixed and identical. We focus on devising approximation algorithm for the problem due to its NP-Completeness.

#### A. NP-hardness of minimum-energy transmission multicast trees

Given a wireless ad hoc network  $M(N, A)$  and an all-to-all multicast session with terminal set  $D \subseteq N$ , the *minimum-energy transmission multicast tree problem* is to construct

a multicast tree  $T_D$  spanning the nodes in  $D$  such that the sum of transmission power of the nodes in  $T_D$  is minimized. Clearly, the sum of transmission power of the nodes in  $T_D$  is the minimum amount of power needed to maintain the topology structure of  $T_D$ , we thus refer to this power setting at each node in  $T_D$  as *maintaining  $T_D$ 's tree topological structure*. In the following we show that this problem is NP-Complete by a reduction from a *minimum Steiner tree problem* that is to find a tree in  $G$  spanning the nodes in  $D$  such that the number of edges in the tree is minimized, given an unweighted, undirected graph  $G(V, E)$  and a terminal set  $D$ . The minimum Steiner tree problem has been shown to be NP-Complete too (ND12) [6].

*Lemma 1:* The minimum-energy transmission multicast tree problem in  $M(N, A)$  is NP-Complete.

*Proof:* We first reduce the minimum Steiner tree problem to the problem of concern as follows. Given an instance  $G(V, E)$  of the minimum Steiner tree problem with an integer  $k'$ , its decision version is to determine whether there is a Steiner tree such that the number of edges in it is no more than  $k'$ . There is a corresponding instance of a wireless ad hoc network  $M(N, A)$  for the minimum-energy transmission multicast tree problem, where  $N = V$  is the set of nodes, and  $(u, v) \in A$  if there is an edge  $(u, v) \in E$ . Each node  $v \in N$  has a fixed transmission power  $t_e$  and zero reception power ( $r_e = 0$ ). Its decision version is to determine whether there is a multicast tree in  $M$  such that the sum of the transmission power of nodes in the tree is no more than  $(k' + 1) * t_e$ , since the number of nodes in any tree is equal to the number of edges in the tree plus one.

We then show that the problem is in NP. Given a multicast tree and a value  $(k' + 1) * t_e$ , to verify whether the tree spans all the nodes in  $D$  and the sum of transmission power of its nodes is no more than  $(k' + 1) * t_e$  can be done within polynomial time. Thus, the minimum-energy transmission multicast tree problem is NP-Complete. ■

### B. Approximation algorithm

The basic idea behind the proposed algorithm is to find a minimum-energy transmission multicast tree  $T_D$ , which is an approximation of the optimal multicast tree  $T_{opt}$  for the minimum-energy all-to-all multicasting problem, while another multicast tree  $T_{app}$  in the communication graph  $G$  derived from the wireless ad hoc network  $M$  is an approximation of  $T_D$ .

Let  $T$  be any multicast tree in  $M$  spanning the nodes in  $D$ . Let  $N_T$  and  $N_T^1$  be the sets of nodes and degree-one nodes in  $T$ . Obviously,  $N_T^1 \subseteq D$ . Assume that  $T$  contains  $n_T = |N_T|$  nodes and  $n_T^1 = |N_T^1|$  degree-one nodes. To realize an all-to-all multicast session using  $T$ , the transmission and reception energy consumptions of a node  $v$  in  $T$  are analyzed as follows.

If  $v \in N_T^1$  is a degree-one node, then its transmission energy consumption is  $l_v * t_e$ . Otherwise, its transmission energy consumption is  $l * t_e$ , because the message originated from every other terminal node must be relayed to the other tree nodes through node  $v$ , and there are  $k$  such messages with total message length  $l$  to be relayed.

If  $v$  is a terminal node in  $D$ , then its reception energy consumption is thus  $(l - l_v) * r_e$  since it receives messages

from all the other terminal nodes except itself. Otherwise, to relay messages of total length  $l$  to the other nodes in  $T$ ,  $v$  requires to receive the messages prior to the relay. Thus, its reception energy consumption is  $l * r_e$ . As a result, the total reception energy consumption of the nodes in  $T$  for the all-to-all multicast session is

$$\begin{aligned} E_T^{receive} &= \sum_{v \in D} (l - l_v) * r_e + l * (n_T - |D|) * r_e \\ &= l * (n_T - 1) * r_e, \end{aligned} \quad (1)$$

and the total energy consumption of realizing an all-to-all multicast session using  $T$  is

$$\mathcal{E}_T = (n_T - n_T^1) * l * t_e + \sum_{v \in N_T^1} l_v * t_e + l * (n_T - 1) * r_e. \quad (2)$$

We now have the following lemma.

*Lemma 2:* Let  $T_D$  be a minimum-energy transmission multicast tree in  $M$ , then, the total energy consumption of realizing an all-to-all multicast session using  $T_D$  is no more than  $k + 1$  times of the optimum, where  $k = |D|$ .

*Proof:* Recall that  $T_{opt}$  is an optimal multicast tree for the minimum-energy all-to-all multicasting problem. Let  $n_{T_D}$  and  $n_{T_{opt}}$  be the number of nodes in  $T_D$  and  $T_{opt}$ . Let  $N_{T_D}^1$  and  $N_{T_{opt}}^1$  be sets of degree-one nodes in  $T_D$  and  $T_{opt}$  with  $n_{T_D}^1 = |N_{T_D}^1|$  and  $n_{T_{opt}}^1 = |N_{T_{opt}}^1|$ . Then, the number of internal nodes in  $T_D$  or  $T_{opt}$  is  $n_{T_D} - n_{T_D}^1$  or  $n_{T_{opt}} - n_{T_{opt}}^1$  respectively. Following Eqn. (2), the total energy consumption of realizing the all-to-all multicast session by using either  $T_D$  or  $T_{opt}$  is either  $\mathcal{E}_{T_D} = (n_{T_D} - n_{T_D}^1) * l * t_e + \sum_{v \in N_{T_D}^1} l_v * t_e + l * (n_{T_D} - 1) * r_e$  or  $\mathcal{E}_{T_{opt}} = (n_{T_{opt}} - n_{T_{opt}}^1) * l * t_e + \sum_{u \in N_{T_{opt}}^1} l(u) * t_e + l * (n_{T_{opt}} - 1) * r_e$ . Clearly,  $\sum_{v \in N_{T_D}^1} l_v \leq l$  and  $\sum_{v \in N_{T_{opt}}^1} l_v \leq l$ , due to  $N_{T_D}^1 \subseteq D$  and  $N_{T_{opt}}^1 \subseteq D$ .

It is not difficult to show that the number of nodes in  $T_D$  is the minimum one among all multicast trees in  $M$ . Otherwise, assume that there is another multicast tree of  $n'$  nodes in  $M$  with  $n' < n_{T_D}$ . Then, the total amounts of transmission energy needed to maintain the topological structure of this latter tree are  $n' * t_e < n_{T_D} * t_e$ , which contradicts that  $T_D$  has the minimum total transmission energy consumption. Therefore,  $n_{T_D} \leq n_{T_{opt}}$ . We thus have

$$\begin{aligned} \frac{\mathcal{E}_{T_D}}{\mathcal{E}_{T_{opt}}} &= \frac{(n_{T_D} - n_{T_D}^1) l t_e + \sum_{v \in N_{T_D}^1} l_v t_e + l (n_{T_D} - 1) r_e}{(n_{T_{opt}} - n_{T_{opt}}^1) l t_e + \sum_{u \in N_{T_{opt}}^1} l(u) t_e + l (n_{T_{opt}} - 1) r_e} \\ &< \frac{(n_{T_D} - n_{T_D}^1) * l + l + l * (n_{T_D} - 1) * r_e / t_e}{(n_{T_{opt}} - n_{T_{opt}}^1) * l + l * (n_{T_{opt}} - 1) * r_e / t_e} \\ &\leq \frac{n_{T_{opt}} - n_{T_D}^1 + 1 + (n_{T_{opt}} - 1) * r_e / t_e}{n_{T_{opt}} - n_{T_{opt}}^1 + (n_{T_{opt}} - 1) * r_e / t_e}, \\ &\quad \text{since } n_{T_D} \leq n_{T_{opt}} \\ &\leq 1 + \frac{n_{T_{opt}}^1 - n_{T_D}^1 + 1}{n_{T_{opt}} - n_{T_{opt}}^1 + (n_{T_{opt}} - 1) * r_e / t_e} \\ &< 1 + \frac{n_{T_{opt}}^1 - n_{T_D}^1 + 1}{n_{T_{opt}} - n_{T_{opt}}^1} \end{aligned}$$

$$\begin{aligned}
&< 1 + \frac{n_{T_{opt}}^1}{n_{T_{opt}} - n_{T_D}^1}, \quad \text{since } n_{T_D}^1 \geq 1 \\
&< 1 + \frac{k}{n_{T_{opt}} - n_{T_{opt}}^1} \\
&< 1 + k, \quad \text{since } n_{T_{opt}}^1 \leq k \ \& \ n_{T_{opt}} - n_{T_{opt}}^1 \geq 1.
\end{aligned}$$

The lemma then follows.  $\blacksquare$

*Theorem 2:* There is an  $O(k(m+n \log n))$  time approximation algorithm for the minimum-energy all-to-all multicasting problem, which delivers a solution within  $2(k+1)$  times of the optimum.

*Proof:* Let  $G(V, E)$  be the communication graph derived from  $M(N, A)$ , where  $V$  is the set of nodes and there is an edge  $(u, v) \in E$  between nodes  $u$  and  $v$  if they are within the transmission range of each other. The weight associated with each node in  $V$  is  $t_e$ . It can be seen that the minimum-energy transmission multicast tree problem in  $M$  is equal to the minimum node-weighted Steiner tree problem in  $G$  that is NP-Complete [8]. Although the best known approximate solution for this latter problem with  $k = |D|$  terminal nodes is  $2 \ln k$  times of the optimum [8], we here deal with one of its special cases where every node has identical weight, and for which, any approximation algorithm for the minimum edge-weighted Steiner tree problem can be applied, since the number of edges in a tree is equal to the number of nodes in it minus one. Thus, an approximate, edge-weighted Steiner tree  $T_{app}$  in  $G$  can be found, using an algorithm by Kou et al [9]. The total energy consumption of realizing an all-to-all multicast session using  $T_{app}$  is no more than twice of that of  $T_D$ , while the total energy consumption of realizing an all-to-all multicast session using  $T_D$  is no more than  $(k+1)$  times of that using  $T_{opt}$ . Thus, there is an approximate solution for the minimum-energy all-to-all multicasting problem, which is within  $2(k+1)$  times of the optimum, since  $\mathcal{E}_{T_D} \leq (k+1)\mathcal{E}_{T_{opt}}$  by Lemma 2.

The dominant running time of the proposed algorithm is the time on the construction of  $T_{app}$ , which takes  $O(k(m+n \log n))$  time for constructing  $k$  single-source shortest path trees.  $\blacksquare$

## V. ALGORITHM FOR ADJUSTABLE TRANSMISSION POWER

In this section we propose an approximation algorithm for the minimum-energy all-to-all multicasting problem where the transmission power of each node is adjustable. Given a wireless ad hoc network  $M = (N, A)$ , a communication graph  $G = (V, E, \gamma)$  is derived from  $M$ , which is a weighted, undirected graph, where  $V = N$ ,  $E = A$ . The weight  $\gamma(u, v)$  assigned to edge  $(u, v) \in E$  is  $d_{u,v}^2$ . A Steiner tree  $T_{opt}^{edge}$  in  $G$  spanning the nodes in  $D$  is a *minimum edge-weighted Steiner tree* if the weighted sum of the edges in it is minimized, where  $D \subseteq V$ .

### A. Approximation algorithm

The idea behind the proposed algorithm is as follows.  $T_{opt}^{edge}$  in  $G(V, E, \gamma)$  is an approximation of  $T_D$  in  $M(N, A)$ , while  $T_D$  is an approximation of  $T_{opt}$  in  $M(N, A)$  for the minimum-energy all-to-all multicasting problem. In the following we detail the proposed approximation algorithm.

*Lemma 3:* If  $T_D$  instead of  $T_{opt}$  is used for realizing an all-to-all multicast session, then the approximate solution is either  $2l/l_{min}$  times of the optimum, or  $2k$  times of the optimum if the message length at each terminal node is identical.

*Proof:* Let  $T$  be a multicast tree of  $M(N, A)$  spanning the nodes in  $D$ . Let  $E_{in}(T)$  and  $E_{out}(T)$  be the sums of transmission power of internal nodes and degree-one nodes in  $T$ , then the total transmission power of nodes in  $T$  is  $E(T) = E_{in}(T) + E_{out}(T)$ . Let  $L_{in}(T)$  and  $L_{out}(T)$  be the total lengths of messages originated from the nodes in  $D$  that serve as internal nodes and degree-one nodes in  $T$ , it is obvious that  $l = L_{in}(T) + L_{out}(T)$ .

Following Eqn. (1), if  $T_D$  or  $T_{opt}$  is used as the shared multicast tree for an all-to-all multicast session, then the total reception energy consumption of receiving messages with total length  $l$  is  $l(n_{T_D} - 1) * r_e$  or  $l(n_{T_{opt}} - 1) * r_e$ , respectively. Let  $w_{T_D}(v)$  and  $w_{T_{opt}}(v)$  be the amounts of transmission power needed at node  $v$  to maintain  $T_D$  and  $T_{opt}$ 's tree topological structures. Recall that  $N_{T_D}^1$  and  $N_{T_{opt}}^1$  are the sets of degree-one nodes in  $T_D$  and  $T_{opt}$ , and  $\mathcal{E}_{T_D}$  or  $\mathcal{E}_{T_{opt}}$  is the total energy consumption of realizing an all-to-all multicast session by using either  $T_D$  or  $T_{opt}$ . Then,  $\mathcal{E}_{T_D} = L_{in}(T_D) * E_{in}(T_D) + (L_{out}(T_D) * E_{in}(T_D) + \sum_{v \in N_{T_D}^1} w_{T_D}(v) * l_v) + l(n_{T_D} - 1) * r_e$  or  $\mathcal{E}_{T_{opt}} = L_{in}(T_{opt}) * E_{in}(T_{opt}) + (L_{out}(T_{opt}) * E_{in}(T_{opt}) + \sum_{v \in N_{T_{opt}}^1} w_{T_{opt}}(v) * l_v) + l(n_{T_{opt}} - 1) * r_e$ .

We show (i)  $E(T_D) \leq E(T_{opt})$  by contradiction. If  $E(T_D) > E(T_{opt})$ , then the total amount of transmission energy needed to maintain  $T_{opt}$ 's tree topological structure is strictly less than that to maintain  $T_D$ 's tree topological structure. This contradicts that  $T_D$  has the minimum transmission energy consumption. Thus,  $E(T_D) \leq E(T_{opt})$ . What followed is to show (ii)  $E(T_D) \geq (n_{T_D} - 1) * r_e$ . Since the minimum transmission energy consumption of each node in  $T_D$  is no less than its reception energy consumption  $r_e$  for a unit-length message transfer, i.e.,  $r_e \leq \min_{v \in N_{T_D}} \{w_{T_D}(v)\}$ . Thus,  $E(T_D) = \sum_{v \in N_{T_D}} w_{T_D}(v) \geq \sum_{v \in N_{T_D}} \min_{v \in N_{T_D}} \{w_{T_D}(v)\} \geq n_{T_D} * r_e > (n_{T_D} - 1) * r_e$ . Let  $l_{min} = \min_{v \in D} \{l_v\}$  and  $l_{max} = \max_{v \in D} \{l_v\}$  be the minimum and maximum message lengths among the  $k$  messages. We have the inequality derivation that appears at the top of the next page.

The lemma then follows.  $\blacksquare$

What followed is to show there is an approximate solution to the minimum-energy transmission multicast tree problem by the following lemma. Let  $EG(T)$  be the edge set of a tree  $T$  in  $G(V, E, \gamma)$ .

*Lemma 4:* Let  $T_D'$  be the corresponding tree of  $T_D$  in the communication graph  $G(V, E, \gamma)$ . Let  $T_{opt}''$  be the corresponding multicast tree in  $M$  derived from tree  $T_{opt}^{edge}$  in  $G(V, E, \gamma)$ . Let  $E(T_{opt}'')$  be the sum of transmission power of the nodes in  $T_{opt}''$ . Then, (i)  $E(T_{opt}'') \leq 2 \sum_{(u,v) \in EG(T_D')} \gamma(u, v)$ ; (ii)  $E(T_D) \geq \sum_{(u,v) \in EG(T_D')} \gamma(u, v)$ ; and (iii)  $E(T_{opt}'') \leq 2E(T_D)$ .

*Proof:* We first show case (i). We notice that the amounts of transmission power of each node  $v \in N_{T_{opt}''}$  in the multicast tree  $T_{opt}''$  are equal to the maximum weight value among the edges incident to  $v$  in  $T_{opt}^{edge}$ , and the weight of any edge

$$\begin{aligned}
\frac{\mathcal{E}_{T_D}}{\mathcal{E}_{T_{opt}}} &= \frac{L_{in}(T_D) * E_{in}(T_D) + (L_{out}(T_D) * E_{in}(T_D) + \sum_{v \in N_{T_D}^1} w_{T_D}(v) * l_v) + l(n_{T_D} - 1) * r_e}{L_{in}(T_{opt}) * E_{in}(T_{opt}) + (L_{out}(T_{opt}) * E_{in}(T_{opt}) + \sum_{v \in N_{T_{opt}}^1} w_{T_{opt}}(v) * l_v) + l(n_{T_{opt}} - 1) * r_e} \\
&\leq \frac{l * E_{in}(T_D) + l_{max} * E_{out}(T_D) + l(n_{T_D} - 1) * r_e}{l * E_{in}(T_{opt}) + l_{min} * E_{out}(T_{opt}) + l(n_{T_{opt}} - 1) * r_e} \\
&\leq \frac{l * E_{in}(T_D) + l * E_{out}(T_D) + l(n_{T_D} - 1) * r_e}{l_{min} * E_{in}(T_{opt}) + l_{min} * E_{out}(T_{opt}) + l(n_{T_{opt}} - 1) * r_e} \\
&= \frac{l * E(T_D) + l(n_{T_D} - 1) * r_e}{l_{min} * E(T_{opt}) + l(n_{T_{opt}} - 1) * r_e} \\
&\leq \frac{l * E(T_D) + l * E(T_D)}{l_{min} * E(T_{opt}) + l(n_{T_{opt}} - 1) * r_e}, \quad \text{since } (n_{T_D} - 1) * r_e \leq E(T_D), \\
&< \frac{2l * E(T_D)}{l_{min} * E(T_{opt})} \leq \frac{2l}{l_{min}}, \quad \text{since } E(T_D) \leq E(T_{opt})
\end{aligned}$$

in  $T_{opt}^{edge}$  can be counted at most twice as the amounts of transmission power of its two endpoints in the calculation of  $E(T_{opt}')$ . Thus,  $E(T_{opt}') \leq 2 \sum_{(u,v) \in EG(T_{opt}^{edge})} \gamma(u,v)$ .

We then show case (ii). Note that  $T_D$  and  $T'_D$  are identical except each edge in  $T'_D$  is assigned a weight while the edge in  $T_D$  is not. For convenience, we assume that  $T'_D$  is a rooted tree and its node set  $N_{T'_D}$  can be partitioned into a number of disjoint subsets  $V_0, V_1, \dots, V_l$  as follows.  $V_0$  is the set of leaf nodes. Initially,  $T_0 = T'_D$ . For any  $i \geq 1$ , denote by  $T_i$  the resulting tree by pruning all leaf nodes from tree  $T_{i-1}$  and  $V_{i-1}$  the set of removed leaf nodes in  $T_{i-1}$ . Keep the removal of leaf nodes from the current tree until the resulting tree contains only the root node. Denote by  $V_i$  the set containing the root node. Then,  $N_{T'_D} = \bigcup_{i=0}^l V_i$  and  $V_i \cap V_j = \emptyset$ ,  $i \neq j$ ,  $1 \leq i, j \leq l$ .

Let  $p(v)$  be the parent of  $v$  and  $C(v)$  the set of child nodes of  $v$  in  $T'_D$ . Then,  $w_{T_D}(v) = \max_{u \in C(v)} \{d_{u,v}^2, d_{v,p(v)}^2\}$  and  $E(T_D) = \sum_{v \in N_{T'_D}} w_{T_D}(v) = \sum_{0 \leq i \leq l} \sum_{v \in V_i} w_{T_D}(v)$ . An edge  $(v, p(v))$  is covered by node  $v$  if it has not been covered yet and  $w_{T_D}(v) \geq \gamma(v, p(v)) = d_{v,p(v)}^2$ . We claim that every tree edge  $(v, p(v))$  in  $T'_D$  is covered by node  $v$  by induction. It is obvious that each tree edge  $(v, p(v))$  incident to a leaf node  $v \in V_0$  is covered by  $v$ , since  $w_{T_D}(v) = d_{v,p(v)}^2$ . We assume that all the tree edges incident to the nodes in  $\bigcup_{j=0}^{i-1} V_j$  are already covered by the nodes in  $\bigcup_{j=0}^{i-1} V_j$ ,  $i \geq 1$ . We now show that a tree edge  $(v, p(v))$  incident to node  $v \in V_i$  is covered by  $v$  in  $V_i$ . If  $w_{T_D}(v) = d_{v,p(v)}^2$ , then, tree edge  $(v, p(v))$  is covered by  $v$ , following the definition of  $w_{T_D}(v)$ ; otherwise,  $w_{T_D}(v) > d_{v,p(v)}^2$  but  $w_{T_D}(v) = \max_{u \in C(v)} \{d_{u,v}^2\}$ . Let  $(u_0, v)$  be the tree edge with  $\gamma(u_0, v) = \max_{u \in C(v)} \{d_{u,v}^2\}$ . Clearly,  $p(u_0) = v$  and the tree edge  $(u_0, v)$  has already been covered by node  $u_0 \in \bigcup_{j=0}^{i-1} V_j$ , following the inductive assumption. Now, the tree edge  $(v, p(v))$  must be covered by  $v$ , because of  $w_{T_D}(v) > d_{v,p(v)}^2$ . This procedure continues until all the edges in  $T'_D$  are covered. Thus,  $E(T_D) = \sum_{v \in N_{T'_D}} w_{T_D}(v) = \sum_{v \in N_{T'_D}} w_{T_D}(v) \geq \sum_{(v,p(v)) \in EG(T'_D)} \gamma(v, p(v)) = \sum_{(u,v) \in EG(T'_D)} \gamma(u, v)$ .

We finally prove case (iii). It is obvious that the weighted sum of the edges in  $T'_D$  is no less than that in  $T_{opt}^{edge}$ , because the latter is the minimum edge-weighted Steiner tree. Thus,

$E(T_{opt}') \leq 2 \sum_{(u,v) \in EG(T_{opt}^{edge})} \gamma(u, v)$  by case (ii), while  $\sum_{(u,v) \in EG(T_{opt}^{edge})} \gamma(u, v) \leq \sum_{(u',v') \in EG(T'_D)} \gamma(u', v')$ . Given  $E(T_D) = \sum_{(u',v') \in EG(T'_D)} \gamma(u', v')$ , we have  $E(T_{opt}') \leq 2E(T_D)$ . ■

We thus have the following theorem.

**Theorem 3:** There is an approximation algorithm for the minimum-energy all-to-all multicasting problem with approximation ratio of  $8l/l_{min}$ . The proposed algorithm takes  $O(k(m + n \log n))$  time.

*Proof:* The approximation ratio of the proposed algorithm is analyzed as follows. There is an approximate solution  $T_{app}$  for  $T_{opt}^{edge}$ , which is twice of the optimum [9], whereas the multicast tree  $T_{opt}''$  in  $M$  is the corresponding tree of  $T_{opt}^{edge}$  in  $G$ , which is an approximate solution of  $T_D$  within twice of the optimum by Lemma 4. Therefore, the approximate solution  $T_{app}$  is four times of the optimum (in terms of  $T_D$ ). In addition,  $T_D$  is an approximation of the optimal multicast tree  $T_{opt}$ , which is  $2l/l_{min}$  times of the optimum by Lemma 3. Thus,  $T_{app}$  is an approximation of  $T_{opt}$ , which is  $8l/l_{min}$  times of the optimum. The dominant running time of the proposed algorithm is the time for finding an approximate solution for the minimum edge-weighted Steiner tree problem, which takes  $O(k(m + n \log n))$  time, by Theorem 2. ■

## VI. DISTRIBUTED IMPLEMENTATION

The proposed centralized algorithms may not be applicable in practice, due to the fact that sometimes it is impossible for each node to have the topological knowledge of the entire network. Instead, each node has only the local knowledge of its neighboring nodes. Based on such a distributed environment, we provide a distributed implementation of the proposed algorithm, which is referred to as algorithm `Dist_Implement` (see Fig. 1). For convenience, we work on the communication graph  $G = (V, E, \gamma)$  instead of the wireless network  $M(N, A)$ .

**Theorem 4:** There is a distributed approximation algorithm for the minimum-energy all-to-all multicasting problem with approximation ratio of  $4kl/l_{min}$ . The proposed algorithm takes  $O(kn)$  time and requires  $O(km)$  messages.

*Proof:*  $T_{app}$  delivered by algorithm `Dist_Implement` is an approximation of  $T_{opt}^{edge}$  within  $k$  times of the optimum,

Algorithm `Dist_Implement` ( $V, E, D, \gamma()$ )

**begin**

1. **for** each node  $v \in D$  **do**
2.     Construct a single source shortest path tree  $T_v$  in  $G$  rooted at  $v$ ;
3.     Prune those branches from  $T_v$  that do not contain nodes in  $D$ , and denote by  $T_v$  the resulting tree if no confusion arises.
4.     Compute the weighted sum of the edges in  $T_v$  and store it at  $v$ .

**endfor**;

5. Find a tree  $T_{v_0}$  rooted at  $v_0 \in D$  from the  $k = |D|$  trees such that the weighted sum of the edges in  $T_{v_0}$  is the minimum. Denote by  $T_{app}$  as  $T_{v_0}$ . Let  $N_{T_{app}}(v)$  be the set of neighboring nodes of  $v$  in  $T_{app}$ .
6. Set the power level of each node  $v$  in  $T_{app}$  by assigning its transmission power to be  $\max_{u \in N_{T_{app}}(v)} \{d_{u,v}^2\}$ .

**end.**

Fig. 1. A distributed algorithm `Dist_Implement`.

while  $T_{opt}^{edge}$  is an approximation of  $T_D$  within twice of the optimum. Furthermore,  $T_D$  is an approximation of the optimal multicast tree  $T_{opt}$ , which is  $2l/l_{min}$  times of the optimum. Therefore,  $T_{app}$  is an approximation of  $T_{opt}$ , which is  $4kl/l_{min}$  times of the optimum.

The computational complexity of algorithm `Dist_Implement` is analyzed as follows. Step 2 takes  $O(n)$  time and requires  $O(m)$  messages by an algorithm due to Chandy and Misra [4]. Step 3 takes  $O(n)$  time and requires  $O(n)$  messages. The implementations of Step 4 takes  $O(n)$  time and  $O(n)$  messages only. The number of iterations from Step 2 to Step 4 is  $k$  ( $= |D|$ ), thus, the total running time for these steps is  $O(kn)$  and the number of messages required is  $O(km)$ . Step 5 can be implemented within  $O(n)$  time using  $O(n)$  messages. The last step takes  $O(n)$  time and requires  $O(n)$  messages. Thus, the distributed algorithm takes  $O(kn)$  time and requires  $O(km)$  messages. ■

## VII. PERFORMANCE EVALUATION

In this section we evaluate the performance of the proposed algorithm against existing algorithms, in terms of total energy consumption of realizing an all-to-all multicast session through experimental simulations. We consider wireless ad hoc networks consisting of 100, 150 and 200 nodes randomly distributed in a  $1,000m \times 1,000m$  region of interest. The maximum transmission range of each node is 250 meters. The fixed transmission power  $t_e$  is set 62,500 ( $t_e^2 = 250^2$ ) units and the reception power  $r_e$  is set 10 units. We assume that the maximum transmission range of each node is no more than 250 meters when its transmission power is adjustable. The length  $l_v$  of a message originated from a terminal node  $v \in D$  is a random integer ranging from 1 to  $10^6$ . In all experiments, the value in each chart is the mean of 100 simulation results performed under 100 randomly network topologies, generated by the NS-2 simulator.

As mentioned in the Introduction, the minimum total energy consumption of realizing an all-to-all multicast session can be achieved if an exclusive routing tree rooted at each terminal node is used to multicast its message to the other terminal nodes. Since finding such an optimal multicast tree is NP-Complete [10], instead, a shortest path tree rooted at each ter-

minal node and spanning the other terminal nodes will be used as the exclusive routing tree at the terminal node. We refer to this multiple multicast trees based shortest path algorithm for realizing all-to-all multicast sessions as algorithm `MMTSP` for short. To evaluate the performance of the proposed algorithms, we will use algorithm `MMTSP` as a performance benchmark to see how far away of the proposed solutions from this optimal one. Without loss of generality, in the rest of this section we use `XXF` and `XXA` to represent the corresponding versions of algorithm `XX` under the models of Fixed transmission power  $t_e$  and Adjustable transmission power, respectively.

### A. Energy overhead on building routing trees

We first investigate the ratio of the energy overhead on constructing a shared routing tree to the total energy consumption of realizing an all-to-all multicast session using the routing tree. To study the energy overhead on constructing a shared routing tree, we conduct experiments by simulation to compute the energy overhead on building the shared routing tree by the proposed distributed algorithm `Dist_Implement`, because it incurs the maximum energy overhead among all the proposed algorithms. We refer to `DISF_C` and `DISA_C` as the energy overheads on building the shared routing tree by algorithm `Dist_Implement` under the models of fixed transmission power and adjustable transmission power respectively.

It can be seen from Fig. 2 (a), (b) and (c) that under different transmission models, the energy overheads on constructing a routing tree is much less than that of realizing an all-to-all multicast session, which is no more than 0.67% or 0.94% of the total energy consumption of realizing an all-to-all multicast session when the transmission power is fixed or adjustable. Therefore, the energy overhead on building a shared routing tree is negligible, in comparison with that of using the routing tree to realize an all-to-all multicast session.

### B. Performance evaluation with fixed transmission power

We then analyze the performance of various algorithms when the transmission power of each node is fixed and identical. We compute the total energy consumption of realizing an

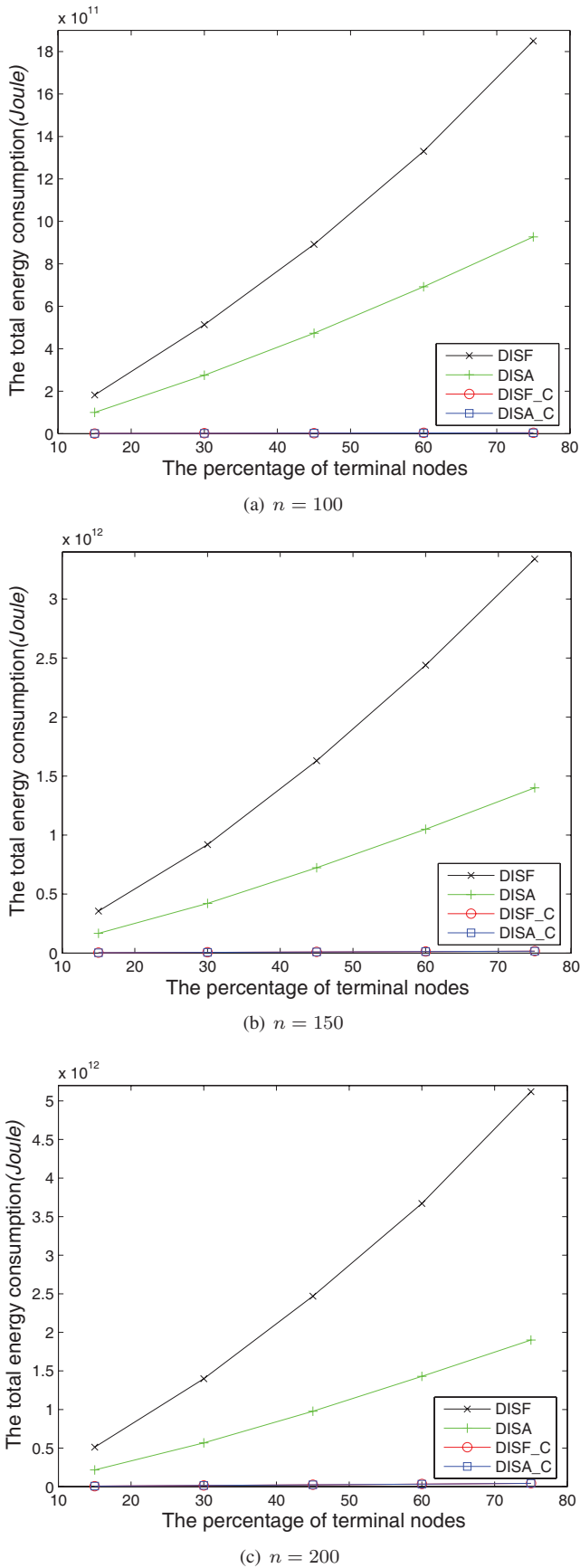


Fig. 2. The energy overhead on building a shared routing tree by the proposed distributed algorithm *Dist\_Implement* for realizing an all-to-all multicast session in an ad hoc network consisting of  $n$  nodes, under the models of both fixed and adjustable transmission power. The percentages of terminal nodes is from 15% to 75% of network nodes.

all-to-all multicast session, by algorithms AAF, DISF, ASTF in [5], LAMF in [7], and SPTF. SPTF is to construct a shortest path tree rooted at a terminal node and spanning all the other nodes in  $D$ , where the distance between two nodes is defined as the minimum number of hops between them. ASTF selects a Rendezvous Point ( $RP$ ) as the tree root and constructs a shortest path tree including all terminal nodes, where the tree root  $RP$  is preferably selected among the nodes with slow mobility. To reduce path costs and distribute traffic more evenly in the network, under certain conditions, a terminal node sends its message to  $RP$  along the shortest path rather than the shared tree path between the terminal node and  $RP$ .  $RP$  then multicasts the message to other terminal nodes, using the shared tree paths. LAMF builds a shared tree centered at a “CORE” node including all the terminal nodes, where the paths from terminal nodes to the CORE are provided by TORA.

Fig. 3 (a), (b) and (c) depict the performance delivered by various algorithms when the number of nodes in the network is 100, 150 and 200 respectively.

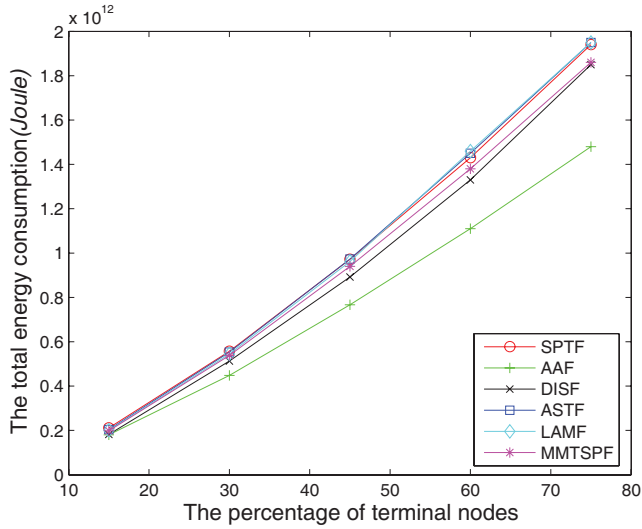
It can be seen from Fig. 3 that approximation algorithm AAF outperforms all the other algorithms that employ a single routing tree. This implies that the use of the shared multicast tree, delivered by algorithm AAF, can substantially save network energy, and the energy savings scale will significantly increase, with the growth of the network size and the percentage of terminal nodes. Meanwhile, it is interesting to see that algorithm AAF always outperforms algorithm MMTSPF, which implies that employing a shared routing tree for realizing all-to-all multicast sessions results in more energy savings than the use of multiple routing trees.

### C. Performance evaluation with adjustable transmission power

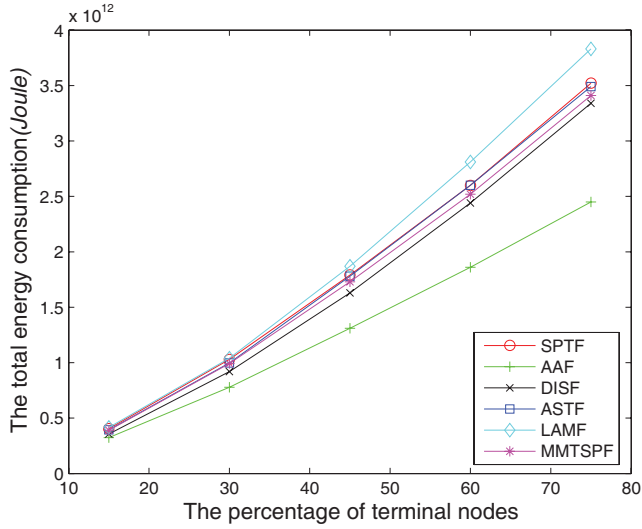
We finally evaluate the performance of different algorithms when the transmission power of each node is adjustable. We compute the total energy consumption of realizing an all-to-all multicast session, by the proposed approximation algorithm AAA and distributed algorithm DISA, algorithms SPTA, ASTA, and LAMA, where SPTA is similar to SPTF, the only difference between them is to find a weighted shortest path tree in this latter one and each edge  $(u, v)$  is assigned a weight of  $d_{u,v}^2$ . ASTA (LAMA) is similar to ASTF (LAMF), the difference between them is that the transmission power of a node  $v$  in the tree is set to be the maximum one among the squares of the distances between  $v$  and its children. Fig. 4 (a), (b) and (c) illustrate the performance of these algorithms when the network consists of 100, 150 and 200 nodes, respectively.

Among the algorithms, the total energy consumption by algorithm AAA is less than that by any of the other algorithms that use the single routing tree significantly. Furthermore, Fig. 4 shows that the total energy consumption by algorithm AAA is almost identical to that by the benchmark algorithm MMTSPA, which implies that the adoption of the shared routing tree for realizing all-to-all multicast sessions is a wise choice.

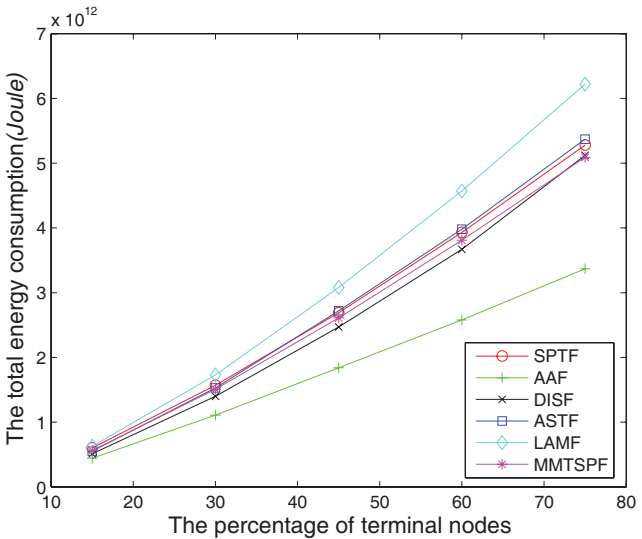




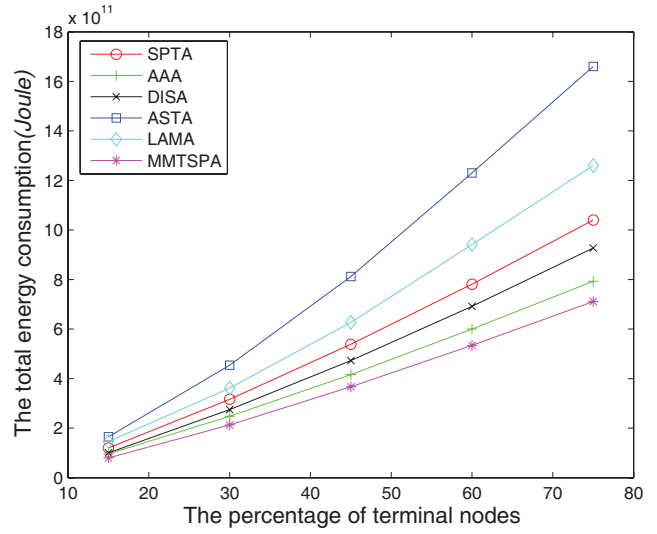
(a)  $n = 100$



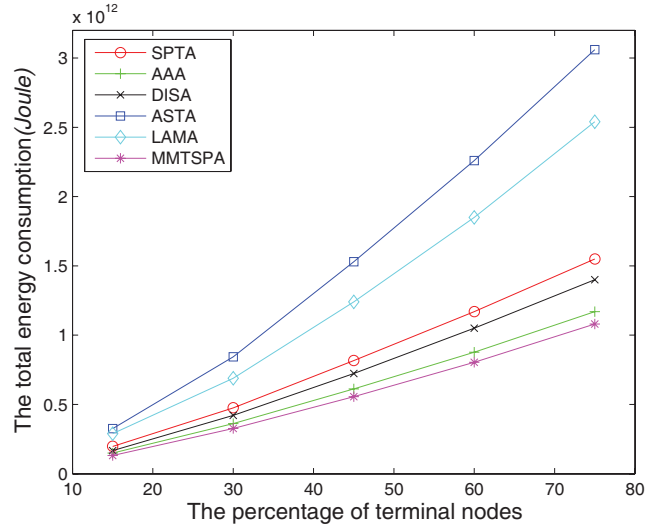
(b)  $n = 150$



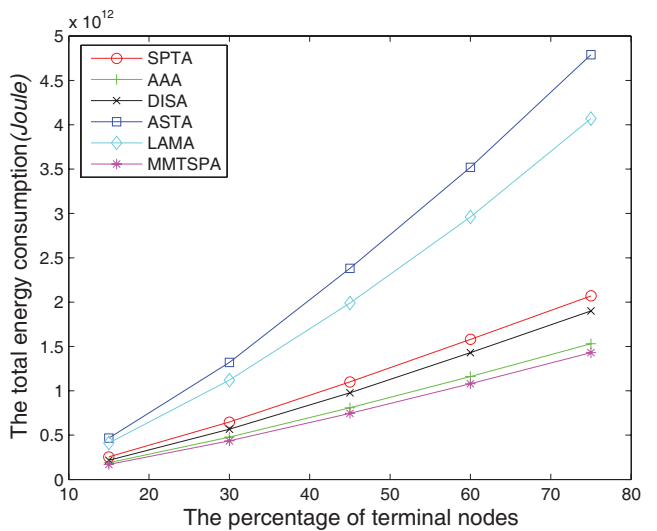
(c)  $n = 200$



(a)  $n = 100$



(b)  $n = 150$



(c)  $n = 200$

Fig. 3. The total energy consumption of realizing all-to-all multicasting by various algorithms in a wireless ad hoc network consisting of  $n$  nodes with identical transmission power. The percentages of terminal nodes is from 15% to 75% of network nodes.

Fig. 4. A wireless ad hoc network consists of  $n$  nodes with adjustable transmission power. The percentage of terminal nodes is from 15% to 75% of network nodes.

### VIII. CONCLUSIONS

In this paper we considered the minimum-energy all-to-all multicasting problem by first showing it is NP-hard. We then devised approximation algorithms with guaranteed approximation ratios. We also provided a distributed implementation of the proposed algorithm. We finally conducted extensive experiments by simulations to evaluate the performance of the proposed algorithm against existing algorithms. The experimental results show the proposed algorithm outperforms existing algorithms significantly in terms of the total energy consumption.

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