IRREGULARITIES IN THE DISTRIBUTION OF PRIMES AND TWIN PRIMES

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Dedicated to Derrick H. Lehmer on the occasion of his 70th birthday

Abstract

The maxima and minima of $\langle L(x)\rangle - \pi(x)$, $\langle R(x)\rangle - \pi(x)$, and $\langle L_2(x)\rangle - \pi_2(x)$ in various intervals up to $x = 8\times 10^{10}$ are tabulated. Here $\pi(x)$ and $\pi_2(x)$ are respectively the number of primes and twin primes not exceeding x, $L(x)$ is the logarithmic integral, $R(x)$ is Riemann's approximation to $\pi(x)$, and $L_2(x)$ is the Hardy-Littlewood approximation to $\pi_2(x)$. The computation of the sum of inverses of twin primes less than 8×10^{10} gives a probable value $1.9021604 \pm 5\times 10^{-7}$ for Brun's constant.

Comments

Only the Abstract is given here. The full paper appeared as [1]. For a more recent evaluation of Brun's constant, which incidentally resulted in the discovery of a bug in the Pentium floating-point divide, see [3].

Errata. On page 49, three lines from the bottom, "17" should be replaced by "16", and "900" by "960". On the same page, six lines from the bottom, "17" should be replaced by "16".

References

[1] R. P. Brent, "Irregularities in the distribution of primes and twin primes", *Mathematics of Computation* (Derrick H. Lehmer special issue) 29 (1975), 43–56. MR 50#1791, 51#5522; Zbl 295.10002. Errata: *ibid* 30 (1976), 198. MR 53#302. See also "Tables concerning irregularities in the distribution of primes and twin primes", UMT 4, *ibid* 29 (1975), 331; and "Tables concerning irregularities in the distribution of primes and twin primes to 10^{11}", UMT 21, *ibid* 30 (1976), 379. rpb024.

[2] R. Sherman Lehman, "On the difference $\pi(x) - \mathrm{li}(x)$", *Acta Arithmetica* 11 (1966), 397–410. MR 34#2546.

[3] T. R. Nicely, "Enumeration to 10^{14} of the Twin Primes and Brun's Constant", *Virginia Journal of Science* 46 (1995), 195–204. For a review see `review04.dvi.gz` .

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