KNUTH'S CONSTANTS TO 1000 DECIMAL AND 1100 OCTAL PLACES

RICHARD P. BRENT

Abstract

In appendices to *The Art of Computer Programming* [3], Knuth lists 33 mathematical constants to 40 decimal and 44 octal places. In this report the same constants are given to 1000 decimal and 1100 octal places. The constants were computed using the MP package [2] on a Univac 1108 computer.

Comments

Only the Abstract is given here. The full report appeared as [1]. Specifically, the constants are

$$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{10}, 2^{1/3}, 3^{1/3}, 2^{1/4}, \phi = (1+\sqrt{5})/2,$$
$$\ln 2, \ln 3, \ln 10, \ln \pi, \ln \phi, 1/\ln 2, 1/\ln 10, 1/\ln \phi,$$
$$\pi, \pi/180, 1/\pi, \pi^2, \sqrt{\pi} = \Gamma(1/2), \Gamma(1/3), \Gamma(2/3),$$
$$e, 1/e, e^2, \gamma, e^\gamma, e^{\pi/4}, \sin 1, \cos 1, \zeta(3), -\ln\ln 2\ .$$

Each constant was computed twice, once with base 10000 and 260 floating-point digits, and once with base 11701 and 250 digits. Each run required about 25 minutes of computer time on a Univac 1108.

The values given in Knuth were verified, with one exception: the 44-digit octal value of $\pi/180$ given in early editions [3] was incorrectly rounded (it should end in 1155, not 1154).

The constants can be computed to arbitrary precision using a program supplied with the MP package [2].

References

[1] R. P. Brent, *Knuth's constants to 1000 decimal and 1100 octal places*, Report TR 47, Computer Centre, ANU (September 1975), 25 pp. See also UMT 30, *Mathematics of Computation* 30 (1976), 668 (reviewed by John W. Wrench, Jr). rpb036

[2] R. P. Brent, Algorithm 524: MP, a Fortran multiple-precision arithmetic package [A1], *ACM Transactions on Mathematical Software* 4 (1978), 71–81. rpb043

[3] D. E. Knuth, *The Art of Computer Programming*, volumes 1–3, Addison-Wesley, Reading, Mass., 1968–1973.

Computer Centre, Australian National University, Canberra, Australia

Key words and phrases. constants, Euler's constant, Gamma function, Knuth, trigonometric functions, zeta(3).

rpb036a typeset using $\mathcal{AMS}$-LaTeX.