COMPUTATION OF THE REGULAR CONTINUED FRACTION FOR EULER'S CONSTANT

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Abstract

We describe a computation of the first $20,000$ partial quotients in the regular continued fraction fo Euler's constant $\gamma = 0.577\ldots$ and $\exp(\gamma) = 1.781\ldots$. A preliminary step was the calculation of γ and $\exp(\gamma)$ to $20,700$D. It follows from the continued fractions that, if γ or $\exp(\gamma)$ is of the form P/Q for integers P and Q, then $|Q| > 10^{10000}$.

Comments

Only the Abstract is given here. The full paper appeared as [2]. The result improved on an earlier (and only partially correct) result of Beyer and Waterman [1]. An interesting connection with the work of Ramanujan is described in [4]. For a sequel which introduced some more efficient algorithms and extended the computation, see [3].

References

1. W. A. Beyer and M. S. Waterman, "Error analysis of a computation of Euler's constant", *Math. Comp.* 28 (1974), 599–604. Also "Decimals and partial quotients of Euler's constant and ln 2", reviewed in RMT 19, *Math. Comp.* 28 (1974), 667.
2. R. P. Brent, "Computation of the regular continued fraction for Euler's constant", *Math. Comp.* 31 (1977), 771–777. MR 55#9490, Zbl 369.10002. See also "γ and $\exp(\gamma)$ to 20700D and their regular continued fractions to 20000 partial quotients", UMT 1, *ibid* 32 (1978), 311. rpb040.
3. R. P. Brent and E. M. McMillan, "Some new algorithms for high-precision computation of Euler's constant", *Math. Comp.* 34 (1980), 305–312. MR 82g:10002. rpb049.
4. R. P. Brent, *An asymptotic expansion inspired by Ramanujan*, Technical Report CMA-MR02-93/SMS-10-93, Centre for Mathematics and its Applications, ANU, February 1993, 7 pp. Also "Ramanujan and Euler's constant", to be presented at *Mathematics of Computation 1943–1993*, Vancouver, August 1993. rpb139.

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