ON THE ZEROS OF THE RIEMANN ZETA FUNCTION IN THE CRITICAL STRIP

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Abstract

We describe a computation which shows that the Riemann zeta function $\zeta(s)$ has exactly 75,000,000 zeros of the form $\sigma + it$ in the region $0 < t < 32{,}585{,}736.4$; all these zeros are simple and lie on the line $\sigma = 1/2$. (A similar result for the first 3,500,000 zeros was established by Rosser, Yohe and Schoenfeld.) Counts of the number of Gram blocks of various types and the number of failures of "Rosser's rule" are given.

Comments

Only the Abstract is given here. The full paper appeared as [1]. For further work, see [2, 3].

References

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1991 *Mathematics Subject Classification.* Primary 11M26; Secondary 11-04, 30-04, 65E05.

Key words and phrases. Gram blocks, Riemann hypothesis, Riemann zeta function, Riemann-Siegel formula, Rosser's rule, Turing's theorem, zeta functions.

The author thanks Enrico Bombieri, John Coates, Harold Edwards, R. Sherman Lehman, Derrick H. Lehmer, Hugh Montgomery, Lowell Schoenfeld and Daniel Shanks for many useful suggestions, and the Australian National University for the provision of computer time.

Received October 4, 1978; revised February 8, 1979.