SOME NEW ALGORITHMS FOR HIGH-PRECISION COMPUTATION OF EULER'S CONSTANT

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Abstract

We describe several new algorithms for the high-precision computation of Euler's constant $\gamma = 0.577\ldots$. Using one of the algorithms, which is based on an identity involving Bessel functions, γ has been computed to 30,100 decimal places. By computing their regular continued fractions, we show that, if γ or $\exp(\gamma)$ is of the form P/Q for integers P and Q, then $|Q| > 10^{15000}$.

Comments

Only the Abstract is given here. The full paper appeared as [2]. For earlier work, see [1]. An interesting connection with the work of Ramanujan is described in [3].

References

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2. R. P. Brent and E. M. McMillan, "Some new algorithms for high-precision computation of Euler's constant", *Math. Comp.* 34 (1980), 305–312. MR 82g:10002. Also Report TR LBL-8729, Lawrence Berkeley Laboratory; and Report TR-CS-79-03, DCS, ANU (January 1979), 16 pp. See also "Euler's constant and its exponential to 30,100 decimals", and "The first 29,000 partial quotients in the regular continued fraction for Euler's constant and its exponential", *Math. Comp.* UMT File. rpb049.
3. R. P. Brent, *An asymptotic expansion inspired by Ramanujan*, Report CMA-MR02-93/SMS-10-93, Centre for Mathematics and its Applications, ANU, February 1993, 7 pp. rpb139.

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