ON THE COMPLEXITY OF COMPOSITION AND GENERALIZED COMPOSITION OF POWER SERIES

R. P. BRENT AND J. F. TRAUB

Abstract

Let $F(x) = f_1x + f_2x^2 + \cdots$ be a formal power series over a field Δ. Let $F^{(0)}(x) = x$ and, for $q = 1, 2, \ldots$, define $F^{(q)}(x) = F^{(q-1)}(F(x))$. The obvious algorithm for computing the first n terms of $F^{(q)}(x)$ is by the composition analogue of repeated squaring. This algorithm has complexity about $\log_2 q$ times that of a single composition. Brent [1] showed that the factor $\log_2 q$ can be eliminated in the computation of the first n terms of $(F(x))^q$ by a change of representation, using the logarithm and exponential functions. We show here that the factor $\log_2 q$ can also be eliminated for the composition problem, unless the complexity of composition is quasi-linear.

$F^{(q)}(x)$ can often, but not always, be defined for more general q. We give algorithms and complexity bounds for computing the first n terms of $F^{(q)}(x)$ whenever it is defined.

We conclude the paper with some open problems.

Comments

Only the Abstract is given here. The full paper appeared as [3]. For related work, see [2].

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