SOME LINEAR-TIME ALGORITHMS FOR SYSTOLIC ARRAYS

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Abstract

We survey some recent results on linear-time algorithms for systolic arrays. In particular, we show how the greatest common divisor (GCD) of two polynomials of degree n over a finite field can be computed in time $O(n)$ on a linear systolic array of $O(n)$ cells; similarly for the GCD of two n-bit binary numbers. We show how n by n Toeplitz systems of linear equations can be solved in time $O(n)$ on a linear array of $O(n)$ cells, each of which has constant memory size (independent of n). Finally, we outline how a two-dimensional square array of $O(n)$ by $O(n)$ cells can be used to solve (to working accuracy) the eigenvalue problem for a symmetric real n by n matrix in time $O(nS(n))$. Here $S(n)$ is a slowly growing function of n; for practical purposes $S(n)$ can be regarded as a constant. In addition to their theoretical interest, these results have potential applications in the areas of error-correcting codes, symbolic and algebraic computations, signal processing and image processing. For example, systolic GCD arrays for error correction have been implemented with the microprogrammable "PSC" chip.

Comments

Only the Abstract is given here. The full paper appeared as [4]. For related work, see [1, 2, 3, 5, 6, 7, 8, 9]. It is conjectured in [8] that $S(n) = O(\log n)$.

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1991 *Mathematics Subject Classification.* Primary 65Y10; Secondary 15A18, 47B35, 65F15, 65H17, 65Y05, 68Q22, 68Q25, 68Q35, 94B99.

Key words and phrases. Systolic arrays, parallel computation, parallel algorithm, greatest common divisor, GCD, Toeplitz system, linear equations, symmetric eigenvalue problem, Hestenes method, Jacobi method, error correction.

rpb079a typeset using $\mathcal{A}\mathcal{M}\mathcal{S}$-LaTeX

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