QR FACTORIZATION OF TOEPLITZ MATRICES

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Abstract

This paper presents a new algorithm for computing the QR factorization of an $m \times n$ Toeplitz matrix in $O(mn)$ operations. The algorithm exploits the procedure for the rank-1 modification and the fact that both principal $(m-1)\times(n-1)$ submatrices of the Toeplitz matrix are identical. An efficient parallel implementation of the algorithm is possible.

Comments

Only the Abstract is given here. The full paper appeared as [1]. For related work, see [2, 3, 4].

The algorithm described in [1] is more stable numerically than that of Sweet [4]. As noted in [2], the factor R in the orthogonal factorization $T = QR$ is computed about as well as would be expected from a Cholesky factorization of $T^T T$. However, the computed Q is not necessarily close to orthogonal.

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