# IMPROVED TECHNIQUES FOR LOWER BOUNDS FOR ODD PERFECT NUMBERS

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## Abstract

If $N$ is an odd perfect number, and $q^k \| N$, $q$ prime, $k$ even, then it is almost immediate that $N > q^{2k}$. We prove here that, subject to certain conditions verifiable in polynomial time, in fact $N > q^{5k/2}$. Using this and related results, we are able to extend the computations in an earlier paper [1] to show that $N > 10^{300}$.

## Comments

Only the Abstract is given here. The full paper appeared as [2]. The main part of the proof that there is no odd perfect number $N$ less than $10^{300}$ is a (very large) tree, each of whose 12655 leaves gives either a contradiction or a sufficiently large lower bound on $N$. The proof tree is available by anonymous ftp in a separate file `rpb116p.dvi.Z` . The integer factorizations used in the proof are also available by anonymous ftp: see [3].

## References

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2. R. P. Brent, G. L. Cohen and H. J. J. te Riele, "Improved techniques for lower bounds for odd perfect numbers", *Mathematics of Computation* 57 (1991), 857–868. MR 92c:11004. Also appeared as Report NM-R8921, Centrum voor Wiskunde en Informatica, Amsterdam, October 1989, 13 pp. A longer version was submitted to the *Mathematics of Computation* UMT file and appeared as Report CMA-R50-89, CMA, ANU, October 1989, 198 pp. rpb116.
3. R. P. Brent, *Factor: An Integer Factorization Program for the IBM PC*, Report TR-CS-89-23, Computer Sciences Laboratory, ANU, October 1989, 7 pp. rpb117.

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Some of the factorizations required for the proof were obtained using the Fujitsu VP-100 of the Australian National University Supercomputer Facility.
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