OPTIMAL ITERATIVE PROCESSES FOR ROOTFINDING

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Abstract

Let $f_0(x)$ be a function of one variable with a simple zero at r_0. An iteration scheme is said to be locally convergent if, for some initial approximation $x_1, \ldots, x_k$ near r_0 and all functions f which are sufficiently close (in a certain sense) to f_0, the scheme generates a sequence $\{x_k\}$ which lies near r_0 and converges to a zero r of f. The order of convergence of the scheme is the infimum of the order of convergence of $\{x_k\}$ for all such functions f. We study iteration schemes which are locally convergent and use only evaluations of $f, f', \ldots, f^{[d]}$ at $x_1, \ldots, x_{k-1}$ to determine x_k, and we show that no such scheme has order greater than $d+2$. This bound is the best possible, for it is attained by certain schemes based on polynomial interpolation.

Comments

Only the Abstract is given here. The full paper appeared as [1] and was reprinted in [3, pages 225–239]. A preliminary version appeared as [2].

Errata

page 328, line 7: $f \varepsilon C^2 \Rightarrow f \in C^2$
page 328, line 11: $f''(r) \neq 0 \Rightarrow f''(r) \neq 0)$
page 329, line 7: which includes $\Rightarrow$ whose centre is
page 332, line 7: $\sum_{j=1}(b_j+1) \Rightarrow \sum_{j=1}^{k-1}(b_j+1)$
page 332, line -9: $S(s,e) \Rightarrow S(a,e)$
page 333, line 14: $xeS \Rightarrow x \in S$
page 333, Lemma 1: $f_0(r_0) = 0 \neq f_0(r_0) \Rightarrow f_0(r_0) = 0 \neq f_0'(r_0)$, e sufficiently small
page 333, (4.5): $xeS \Rightarrow x \in S$
page 334, line 5: $S(t_0,e) \Rightarrow S(r_0,e)$
page 336, (5.15): $(x_k - x_j)^b \Rightarrow (x_k - x_j)^{b_j}$
page 339, (6.7): $h-1 \Rightarrow k-1$
page 340, line 4 of §VII: calues $\Rightarrow$ values

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Key words and phrases. Root-finding, zero-finding, analytic computational complexity, iteration schemes, order of convergence, interpolation.

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References

[1] R. P. Brent, S. Winograd and P. Wolfe, “Optimal iterative processes for rootfinding”, *Numerische Mathematik* 20 (1973), 327–341. CR 15#26753, MR 47#6079. rpb016.

[2] R. P. Brent, S. Winograd and P. Wolfe, *Optimal Iterative Processes for Rootfinding*, Report RC3960, IBM T. J. Watson Research Laboratory, Yorktown Heights, New York, August 1972, 29 pp.

[3] R. P. Brent, *Topics in computational complexity and the analysis of algorithms*, Report TR-CS-80-14, DCS, ANU, October 1980, 375 pp. (D. Sc. thesis). rpb062.

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