THE PARALLEL EVALUATION OF GENERAL ARITHMETIC EXPRESSIONS

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Abstract

It is shown that arithmetic expressions with $n \geq 1$ variables and constants; operations of addition, multiplication, and division; and any depth of parenthesis nesting can be evaluated in time $4\log_2 n + 10(n-1)/p$ using $p \geq 1$ processors which can independently perform arithmetic operations in unit time. This bound is within a constant factor of the best possible. A sharper result is given for expressions without the division operation, and the question of numerical stability is discussed.

Comments

Only the Abstract is given here. The full paper appeared as [2] and was reprinted in [3, pages 39–44]. For earlier work see [1].

The result of [2] is the best possible, up to small constant factors. It should be compared with the well-known result of Valiant *et al* [4] that any multivariate polynomial of degree d which can be evaluated sequentially in n steps can be evaluated in $O(\log d)(\log n + \log d)$ parallel steps using $O(n^3 d^6)$ processors. The latter result is more general, but weaker, because of the $\log d$ terms in the time bound and the much larger number of processors. Both results apply to Boolean expressions, so they have implications for the design of circuits with small depth, i.e. small delay.

References

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