CONCERNING $\int_0^1 \cdots \int_0^1 (\mathbf{x_1}^2 + \cdots + \mathbf{x_k}^2)^{1/2} \mathbf{dx_1 \ldots dx_k}$ AND A TAYLOR SERIES METHOD

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Abstract

The integral of the title equals the mean distance m_k from the origin of a point uniformly distributed over the k-dimensional unit hypercube I^k. Closed form expressions are given for $k = 1, 2$ and 3, while for general k, $m_k \simeq (k/3)^{1/2}$. Using *inter alia* methods from geometry, Cauchy-Schwarz inequalities and Taylor series expansions, several inequalities and an asymptotic series for m_k are established. The Taylor series method also yields a slowly converging infinite series for m_k and can be applied to more general problems including the mean distance between two points independently distributed at random in I^k.

Comments

Only the Abstract is given here. The full paper appeared as [1]. The first three values of m_k are

$$m_1 = 1/2,$$
$$m_2 = \left(\sqrt{2} + \ln(1+\sqrt{2})\right)/3 \simeq 0.76519572,$$
$$m_3 = \sqrt{3}/4 + \ln\left((1+\sqrt{3})/\sqrt{2}\right) - \pi/24 \simeq 0.96059196.$$

References

[1] R. S. Anderssen, R. P. Brent, D. J. Daley and P. A. P. Moran, "Concerning $\int_0^1 \cdots \int_0^1 (x_1{}^2 + \cdots + x_k{}^2)^{1/2} dx_1 \ldots dx_k$ and a Taylor series method", *SIAM J. Applied Mathematics* 30 (1976), 22–30. MR 52#15773, Zbl 337.65022.

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