

The Complexity of Computational Problem Solving

Edited by
R. S. Anderssen and R. P. Brent



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TABLE OF CONTENTS

	Introduction	1
R.S. Anderssen	Introductory remarks	3
L.J. Croves	The complexity of programming	17
R.B. Stanton	Complexity in program structures	32
W.B. Croft	The efficiency of information retrieval	47
C.J. van Rijsbergen		
D.J. Kuck	On the speedup and cost of parallel computation	63
C.K. Yuen	Fast methods for performing convolutions	79
A. Cantoni	The design of adaptive filters	89
R.P. Brent	The complexity of multiple-precision arithmetic	126
B.J. Omodei	The effect of the choice of coordinate functions on the cost of solving elliptic boundary value problems	166
G. Havas	Some complexity problems in algebraic computations	184
A.J. Jakeman	Fast computational procedures based on the FFT	193
M.A. Saunders	The complexity of LU updating in the simplex method	214
D.H. Anderson	An efficient algorithm for discrete, nonlinear, best approximation problems	231
S. Woodford	The Modified Givens' transformations	241
D.M. Ryan, B.A. Foster and I.M. Cheshire	The complexity of real-world scheduling problems	248

FOREWORD

This publication reports the proceedings of a seminar on 'The complexity of computational problem solving', held at the Australian National University on Wednesday, 11 December, 1974. Papers dealing with both theoretical and practical aspects of computational problem solving were classified under the following three broad headings:

1. Problem formulation and programming,
2. Hardware, and
3. Computational methods.

The first classification included papers on structured programming, the complexity of program structures, and information retrieval. The second included papers on multiprocessor machines, fast hardware implementation of convolutions, and the design of adaptive filters. The third and most diverse classification included papers on multiple-precision arithmetic, boundary value problems, finite group computations, the fast Fourier transform, numerically stable implementation of the simplex method, nonlinear best approximation, fast Givens' transformations, and large scheduling problems.

The organizers take this opportunity to record their appreciation of the contributions of the speakers and co-authors, the chairman (Dr M.R. Osborne of the ANU, Professor R.W. Robinson of the University of Newcastle, Professor D.J. Kuck of the University of Illinois, and Dr John Myhill of the Canberra College of Advanced Education), and the participants.

The excellent work of Mr Keith Burrows in tracing the figures and Mrs Barbara Geary in typing the papers should be evident to the reader.

R.S. Anderssen,

R.P. Brent.

INTRODUCTION

The aim of the seminar was the examination of "complexity" in its broader rather than its narrower context. Strictly speaking, the subject "computational complexity" is concerned with upper and lower bounds on the time required to perform various computations, given certain mathematical models of computing machines. However, a broader interpretation of "complexity" includes the study of all aspects of efficiency in computational problem solving, from the preliminary formulation, programming and debugging stages, through to the final stage of interpreting the results obtained.

The papers presented cover different aspects of this broader interpretation of "complexity". The first four papers look at questions connected with problem formulation, programming, and the computer manipulation of information.

In his introduction, Anderssen discusses complexity considerations related to the choice of the problem formulation, as well as making some introductory remarks about the nature of complexity. Stanton and Groves discuss various aspects of the complexity of programming. Groves concentrates on general programming questions, while Stanton concentrates on aspects of structured programming. In the fourth talk, Croft examines the efficiency problems which arise in information retrieval and discusses the progress which is being made towards their solution.

The three talks by Kuck, Yuen and Cantoni concentrate on hardware. Kuck examines the various complexity considerations which arise, if parallel rather than sequential processing is used. Yuen discusses the different strategies involved, if hardware rather than software is used to implement certain computational procedures which are used repeatedly. Finally, Cantoni shows that, when comparing different hardware designs for the same application, the actual cost of implementation becomes a prime consideration which can override all other efficiency questions.

The remaining eight papers treat various complexity aspects of computational methods. Brent and Havas look at questions which have a theoretical flavour. Havas examines some complexity problems which arise in algebraic computations, while Brent looks at the complexity of multiple-precision arithmetic. In the remaining six papers, more practical questions are treated. Omodei examines the effect various choices of coordinate functions have on the cost of variational methods and discusses the trade-off which exists between stability and various strategies of choice. Jakeman examines various ways in which the fast Fourier transform can be used to implement faster algorithms than the standard for complex computational problems, such as numerical quadrature, differentiation, and the solution of integral equations. Woodford compares

the efficiencies of some algorithms for performing plane rotations.

The last three talks are concerned with various aspects of linear and non-linear programming. Saunders considers large-scale linear programming and suggests a scheme for implementing the method of Bartels and Golub in a virtual memory environment. Anderson discusses the use of linear programming procedures in the development of efficient methods for best approximation, and, finally, Ryan discusses a real-world application by examining the optimal solution of vehicle scheduling problems.

R.S. Anderssen

R.P. Brent