A GUIDE TO THE LITERATURE ON ALGEBRAIC STACKS

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ABSTRACT. We provide an informal guide to useful books and research papers on algebraic stacks. It is undoubtedly incomplete.

Any comments, corrections, changes, additions, or suggestions are welcome!

1. Short introductory articles

- Barbara Fantechi: *Stacks for Everybody* [Fan01]
- Dan Edidin: *What is a stack?* [Edi03]
- Dan Edidin: *Notes on the construction of the moduli space of curves* [Edi00]
- Angelo Vistoli: *Intersection theory on algebraic stacks and on their moduli spaces*, and especially the appendix. [Vis89]

2. Classic references

- Mumford: *Picard groups of moduli problems* [Mum65]

    Mumford never uses the term "stack" here but the concept is implicit in the paper; he computes the picard group of the moduli stack of elliptic curves.

- Deligne, Mumford: *The irreducibility of the space of curves of given genus* [DM69]

    This influential paper introduces "algebraic stacks" in the sense which are now universally called Deligne-Mumford stacks (stacks with representable diagonal which admit étale presentations by schemes). There are many foundational results *without proof*. The paper uses stacks to give two proofs of the irreducibility of the moduli space of curves of genus g.

- Artin: *Versal deformations and algebraic stacks* [Art74]

    This paper introduces "algebraic stacks" which generalize Deligne-Mumford stacks and are now commonly referred to as *Artin stacks*, stacks with representable diagonal which admit smooth presentations by schemes.[1] This paper gives deformation-theoretic criterion known as Artin's criterion which allows one to prove that a given moduli stack is an Artin stack without explicitly exhibiting a presentation.

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[1]The terms "Artin stack" and "algebraic stack" are often used interchangeably and there does not seem to be a universal agreement on the conditions for the diagonal for either of these terms. For instance, [LMB00] assumes throughout a separated and quasi-compact diagonal while in [STACKS] and [Ols07a], these hypotheses are only assumed when necessary. Additionally, in this paper, Artin assumes that the stacks are locally of finite type over a base scheme which if finite type over an excellent Dedekind ring.

3. Books and online notes

- Laumon, Moret-Bailly: *Champs Algebriques* [LMB00]
  This book is currently the most exhaustive reference on stacks containing many foundational results. It assumes the reader is familiar with algebraic spaces and frequently references Knutson's book [Knu71]. There is an error in chapter 12 concerning the functoriality of the lisse-étale site of an algebraic stack. One doesn't need to worry about this as the error has been patched by Martin Olsson (see [Ols07b]) and the results in the remaining chapters (after perhaps slight modification) are correct.

- The stacks project authors: *The stacks project* [STACKS].
  This work in progress is an open source textbook on algebraic stacks started by Johan de Jong. This book (currently 1326 pages!) develops the necessary algebraic geometry, commutative algebra and set theory necessary for the theory of algebraic stacks currently ending with the definition of an algebraic stack. Expect more material on algebraic stacks soon!

- Anton Geraschenko: *Lecture notes for Martin Olsson's class on stacks* [Ols07a]
  This course systematically develops the theory of algebraic spaces before introducing algebraic stacks (first defined in Lecture 27!). In addition to basic properties, the course covers the equivalence between being Deligne-Mumford and having unramified diagonal, the lisse- etale site on an Artin stack, the theory of quasi-coherent sheaves, the Keel-Mori theorem, cohomological descent, and gerbes (and their relation to the Brauer group). There are also some exercises.

- Behrend, Conrad, Edidin, Fantechi, Fulton, Göttsche, and Kresch: *Algebraic stacks*, online notes for a book being currently written [BCE$^+$]
  The aim of this book is to give a friendly introduction to stacks without assuming a sophisticated background with a focus on examples and applications. Unlike [LMB00], it is not assumed that the reader has digested the theory of algebraic spaces. Instead, Deligne-Mumford stacks are introduced with algebraic spaces being a special case with part of the goal being to develop enough theory to prove the assertions in [DM69]. The general theory of Artin stacks is to be developed in the second part. Only a fraction of the book is now available on Kresch's website.

4. Related references on foundations of stacks

- Vistoli: *Notes on Grothendieck topologies, fibered categories and descent theory* [Vis05]
  Contains useful facts on fibered categories, stacks and descent theory in the fpqc topology as well as rigorous proofs.

- Knutson: *Algebraic Spaces* [Knu71]
  This book, which evolved from his PhD thesis under Michael Artin, contains the foundations of the theory of algebraic spaces. The book [LMB00] frequently references this text. See also Artin's papers on algebraic spaces: [Art69a], [Art69b],[Art69c], [Art70],[Art71a], [Art71b], [Art73], and [Art74]

- Grothendieck et al, *Theorie de Topos et Cohomologie Etale des Schemas I, II, III* also known as SGA4 [SGA4]

     Volume 1 contains many general facts on universes and sites. Grothendieck's expose [SGA1, Expose VI] discusses fibered categories. Deligne's Expose XVIII in SGA4 introduces Picard stacks.

- Jean Giraud: *Cohomologie non abelienne* [Gir65]

     The book discusses fibered categories, stacks, torsors and gerbes over general sites but does not discuss algebraic stacks. For instance, if G is a sheaf of abelian groups on X, then in the same way $H^1(X, G)$ can be identified with G-torsors, $H^2(X, G)$ can be identified with an appropriately defined set of G-gerbes. When G is not abelian, then $H^2(X, G)$ is defined as the set of G-gerbes.

- Kelly and Street: *Review of the elements of 2-categories* [KS74]

     The category of stacks form a 2-category although a simple type of 2-category where are 2-morphisms are invertible. This is a reference on general 2-categories. I have never used this so I cannot say how useful it is. Also note that [STACKS] contains some basics on 2-categories.

5. Papers in the literature

Below is a list of research papers which contain fundamental results on stacks and algebraic spaces. The intention of the summaries is to indicate only the results of the paper which contribute toward stack theory; in many cases these results are subsidiary to the main goals of the paper. We divide the papers into categories with some papers falling into multiple categories.

5.1. Deformation theory and algebraic stacks. The first three papers by Artin do not contain anything on stacks but they contain powerful results with the first two papers being essential for [Art74].

- Artin: *Algebraic approximation of structures over complete local rings* [Art69a]

     It is proved that under mild hypotheses any effective formal deformation can be approximated: if $F : (Sch/S) \to (Sets)$ is a contravariant functor locally of finite presentation with S finite type over a field or excellent DVR, $s \in S$, and $\hat{\xi} \in F(\hat{\mathcal{O}}_{S,s})$ is an effective formal deformation, then for any $n > 0$, there exists an residually trivial étale neighborhood $(S', s') \to (S, s)$ and $\xi' \in F(S')$ such that ξ' and $\hat{\xi}$ agree up to order n (ie. have the same restriction in $F(\mathcal{O}_{S,s}/\mathfrak{m}^n)$).

- Artin: *Algebraization of formal moduli I* [Art69b]

     It is proved that under mild hypotheses any effective formal versal deformation is algebraizable. Let $F : (Sch/S) \to (Sets)$ be a contravariant functor locally of finite presentation with S finite type over a field or excellent DVR, $s \in S$ be a locally closed point , $\hat{A}$ be a complete noetherian local $\mathcal{O}_S$-algebra with residue field k' a finite extension of $k(s)$, and $\hat{\xi} \in F(\hat{A})$ be an effective formal versal deformation of an element $\xi_0 \in F(k')$. Then there is a scheme X finite type over S and a closed point $x \in X$ with residue field $k(x) = k'$ and an element $\xi \in F(X)$ such that there is an isomorphism $\hat{\mathcal{O}}_{X,x} \cong \hat{A}$ identifying the restrictions of ξ and $\hat{\xi}$ in each $F(\hat{A}/\mathfrak{m}^n)$. The algebraization is unique if $\hat{\xi}$ is a universal

deformation. Applications are given to the representability of the Hilbert and Picard schemes.

- Artin: *Algebraization of formal moduli. II* [Art70]

   Vaguely, it is shown that if one can contract a closed subset $Y' \subseteq X'$ formally locally around Y', then exists a global morphism $X' \to X$ contracting Y with X an algebraic space.

- Artin: *Versal deformations and algebraic stacks* [Art74]

   This momentous paper builds on his work in [Art69a] and [Art69b]. This paper introduces Artin's criterion which allows one to prove algebraicity of a stack by verifying deformation-theoretic properties. More precisely (but not very precisely), Artin constructs a presentation of a limit preserving stack $\mathcal{X}$ locally around a point $x \in \mathcal{X}(k)$ as follows: assuming the stack $\mathcal{X}$ satisfies Schlessinger's criterion([Sch68]), there exists a formal versal deformation $\hat{\xi} \in \varprojlim \mathcal{X}(\hat{A}/\mathfrak{m}^n)$ of x. Assuming that formal deformations are effective (ie. $\mathcal{X}(\hat{A}) \to \varprojlim \mathcal{X}(\hat{A}/\mathfrak{m}^n)$ is bijective), then one obtains an effective formal versal deformation $\xi \in \mathcal{X}(\hat{A})$. Using results in [Art69b], one produces a finite type scheme U and an element $\xi_U : U \to \mathcal{X}$ which is formally versal at a point $u \in U$ over x. Then if we assume $\mathcal{X}$ admits a deformation and obstruction theory satisfying certain conditions (ie. compatibility with étale localization and completion as well as constructibility condition), then it is shown in section 4 that formal versality is an open condition so that after shrinking U, $U \to \mathcal{X}$ is smooth. Artin also presents a proof that any stack admitting an fppf presentation by a scheme admits a smooth presentation by a scheme so that in particular one can form quotient stacks by flat, separated, finitely presented group schemes.

- Conrad, de Jong: *Approximation of Versal Deformations* [CdJ02]

   This paper offers an approach to Artin's algebraization result by applying Popescu's powerful result: if A is a noetherian ring and B a noetherian A-algebra, then the map $A \to B$ is a regular morphism if and only if B is a direct limit of smooth A-algebras. It is not hard to see that Popescu's result implies Artin's approximation over an arbitrary excellent scheme (the excellence hypothesis implies that for a local ring A, the map $A^{\mathrm{h}} \to \hat{A}$ from the henselization to the completion is regular). The paper uses Popescu's result to give a "groupoid" generalization of the main theorem in [Art69b] which is valid over arbitrary excellent base schemes and for arbitrary points $s \in S$. In particular, the results in [Art74] hold under an arbitrary excellent base. They discuss the étale-local uniqueness of the algebraization and whether the automorphism group of the object acts naturally on the henselization of the algebraization.

- Starr: *Artin's axioms, composition, and moduli spaces* [Sta06]

   The paper establishes that Artin's axioms for algebraization are compatible with the composition of 1-morphisms.

- Martin Olsson: *Deformation theory of representable morphism of algebraic stacks* [Ols06a]

This generalizes standard deformation theory results for morphisms of schemes to representable morphisms of algebraic stacks in terms of the cotangent complex. These results cannot be viewed as consequences of Illusie's general theory as the cotangent complex of a representable morphism $X \to \mathcal{X}$ is not defined in terms of cotangent complex of a morphism of ringed topoi (because the lisse-étale site is not functorial).

5.2. Coarse moduli spaces.

- Keel, Mori: *Quotients in Groupoids* [KM97]

    It had apparently long been "folklore" that separated Deligne-Mumford stacks admitted coarse moduli spaces. A rigorous (although terse) proof of the following theorem is presented here: if $\mathcal{X}$ is an Artin stack locally of finite type over a noetherian base scheme such that the inertia stack $I_{\mathcal{X}} \to \mathcal{X}$ is finite, then there exists a coarse moduli space $\phi : \mathcal{X} \to Y$ with ϕ separated and Y an algebraic space locally of finite type over S. The hypothesis that the inertia is finite is precisely the right condition: there exists a coarse moduli space $\phi : \mathcal{X} \to Y$ with ϕ separated if and only if the inertia is finite.

- Conrad: *The Keel-Mori Theorem via Stacks* [Con05]

    Keel and Mori's paper [KM97] is written in the groupoid language and some find it challenging to grasp. Brian Conrad presents a stack-theoretic version of the proof which is quite transparent although it uses the sophisticated language of stacks. Conrad also removes the noetherian hypothesis.

- Rydh: *Existence of quotients by finite groups and coarse moduli spaces* [Ryd07]

    Rydh removes the hypothesis from [KM97] and [Con05] that $\mathcal{X}$ be finitely presented over some base.

- Abramovich, Olsson, Vistoli: *Tame stacks in positive characteristic* [AOV08]

    They define a *tame Artin stack* as an Artin stack with finite inertia such that if $\phi : \mathcal{X} \to Y$ is the coarse moduli space, ϕ_* is exact on quasi-coherent sheaves. They prove that for an Artin stack with finite inertia, the following are equivalent: $\mathcal{X}$ is tame $\iff$ the stabilizers of $\mathcal{X}$ are linearly reductive $\iff$ $\mathcal{X}$ is étale locally on the coarse moduli space a quotient of an affine scheme by a linearly reductive group scheme. For a tame Artin stack, the coarse moduli space is particularly nice. For instance, the coarse moduli space commutes with arbitrary base change while a general coarse moduli space for an Artin stack with finite inertia will only commute with flat base change.

- Alper: *Good moduli spaces for Artin stacks* [Alp08]

    For general Artin stacks with infinite affine stabilizer groups (which are necessarily non-separated), coarse moduli spaces often do not exist. The simplest example is $[\mathbf{A}^1/\mathbf{G}_m]$. It is defined here that a quasi-compact morphism $\phi : \mathcal{X} \to Y$ is a *good moduli space* if $\mathcal{O}_Y \to \phi_*\mathcal{O}_{\mathcal{X}}$ is an isomorphism and ϕ_* is exact on quasi-coherent sheaves. This notion generalizes a tame Artin stack in [AOV08] as well as encapsulates Mumford's geometric invariant theory: if G is a reductive group acting linearly on $X \subseteq \mathbf{P}^n$, then the morphism from the quotient stack of the semi-stable locus to the GIT quotient $[X^{ss}/G] \to X//G$ is a good moduli space. The notion of a

good moduli space has many nice geometric properties: (1) ϕ is surjective, universally closed, and universally submersive, (2) ϕ identifies points in Y with points in $\mathcal{X}$ up to closure equivalence, (3) ϕ is universal for maps to algebraic spaces, (4) good moduli spaces are stable under arbitrary base change, and (5) a vector bundle on an Artin stack descends to the good moduli space if and only if the representations are trivial at closed points.

5.3. Intersection theory.

- Vistoli: *Intersection theory on algebraic stacks and on their moduli spaces* [Vis89]
  This paper develops the foundations for intersection theory with rational coefficients for Deligne-Mumford stacks. If $\mathcal{X}$ is a separated Deligne-Mumford stack, the chow group $A_*(\mathcal{X})$ with rational coefficients is defined as the free abelian group of integral closed substacks of dimension k up to rational equivalence. There is a flat pullback, a proper push-forward and a generalized Gysin homomorphism for regular local embeddings. If $\phi : \mathcal{X} \to Y$ is a moduli space (ie. a proper morphism with is bijective on geometric points), there is an induced push-forward $A_*(\mathcal{X}) \to A_k(Y)$ which is an isomorphism.

- Edidin, Graham: *Equivariant Intersection Theory* [EG98]
  The purpose of this article is to develop intersection theory with integral coefficients for a quotient stack $[X/G]$ of an action of an algebraic group G on an algebraic space X or, in other words, to develop a G-equivariant intersection theory of X. Equivariant chow groups defined using only invariant cycles does not produce a theory with nice properties. Instead, generalizing Totaro's definition in the case of BG and motivated by the fact that if $V \to X$ is a vector bundle then $A_i(X) \cong A_i(V)$ naturally, the authors define $A_i^G(X)$ as follows: Let $\dim(X) = n$ and $\dim(G) = g$. For each i, choose a l-dimensional G-representation V where G acts freely on an open subset $U \subseteq V$ whose complement as codimension $d > n - i$. So $X_G = [X \times U/G]$ is an algebraic space (it can even be chosen to be a scheme). Then they define $A_i^G(X) = A_{i+l-g}(X_G)$. For the quotient stack, one defines $A_i([X/G]) := A_{i+g}^G(X) = A_{i+l}(X_G)$. In particular, $A_i([X/G]) = 0$ for $i > \dim[X/G] = n - g$ but can be non-zero for $i << 0$ (eg. $A_i(B\mathbf{G}_m) = \mathbf{Z}$ for $i \leq 0$). They establish that these equivariant Chow groups enjoy the same functorial properties as ordinary Chow groups. Furthermore, they establish that if $[X/G] \cong [Y/H]$ that $A_i([X/G]) = A_i([Y/H])$ so that the definition is independent on how the stack is presented as a quotient stack.

- Kresch: *Cycle Groups for Artin Stacks* [Kre99]
  Kresch defines Chow groups for arbitrary Artin stacks agreeing with Edidin and Graham's definition in [EG98] in the case of quotient stack. For algebraic stacks with affine stabilizer groups, the theory satisfies the usual properties.

- Behrend and Fantechi: *The intrinsic normal cone* [BF97]
  Generalizing a construction due to Li and Tian, Behrend and Fantechi construct a virtual fundamental class for a Deligne-Mumford stack.

5.4. Quotient stacks. Quotient stacks[2] form a very important subclass of Artin stacks which include almost all moduli stacks studied by algebraic geometers. The geometry of a quotient stack $[X/G]$ is the G-equivariant geometry of X. It is often easier to show properties are true for quotient stacks and some results are only known to be true for quotient stacks. The following papers address: When is an algebraic stack a global quotient stack? Is an algebraic stack "locally" a quotient stack?

- Laumon, Moret-Bailly: [LMB00, Chapter 6]

    Chapter 6 contains several facts about the local and global structure of algebraic stacks. It is proved that an algebraic stack $\mathcal{X}$ over S is a quotient stack $[Y/G]$ with Y an algebraic space (resp. scheme, resp. affine scheme) and G a finite group if and only if there exists an algebraic space (resp. scheme, resp. affine scheme) Y' and an finite étale morphism $Y' \to \mathcal{X}$. It is shown that any Deligne-Mumford stack over S and $x : \mathrm{Spec} K \to \mathcal{X}$ admits an representable, étale and separated morphism $\phi : [X/G] \to \mathcal{X}$ where G is a finite group acting on an affine scheme over S such that $\mathrm{Spec} K = [X/G] \times_{\mathcal{X}} \mathrm{Spec} K$. The existence of presentations with geometrically connected fibers is also discussed in detail.

- Edidin, Hassett, Kresch, Vistoli: *Brauer Groups and Quotient stacks* [EHKV01]

    First, they establish some fundamental (although not very difficult) facts concerning when a given algebraic stack (always assumed finite type over a noetherian scheme in this paper) is a quotient stack. For an algebraic stack $\mathcal{X}$: $\mathcal{X}$ is a quotient stack $\iff$ there exists a vector bundle $V \to \mathcal{X}$ such that for every geometric point, the stabilizer acts faithfully on the fiber $\iff$ there exists a vector bundle $V \to \mathcal{X}$ and a locally closed substack $V^0 \subseteq V$ such that V^0 is representable and surjects onto F. They establish that an algebraic stack is a quotient stack if there exists finite flat cover by an algebraic space. Any smooth Deligne-Mumford stack with generically trivial stabilizer is a quotient stack. They show that a $\mathbf{G}_m$-gerbe over a noetherian scheme X corresponding to $\beta \in H^2(X, \mathbf{G}_m)$ is a quotient stack if and only if β is in the image of the Brauer map $\mathrm{Br}(X) \to \mathrm{Br}'(X)$. They use this to produce a non-separated Deligne-Mumford stack that is not a quotient stack.

- Totaro: *The resolution property for schemes and stacks* [Tot04]

    A stack has the resolution property if every coherent sheaf is the quotient of a vector bundle. The first main theorem is that if $\mathcal{X}$ is a normal noetherian algebraic stack with affine stabilizer groups at closed points, then the following are equivalent: (1) $\mathcal{X}$ has the resolution property and (2) $\mathcal{X} = [Y/\mathrm{GL}_n]$ with Y quasi-affine. In the case $\mathcal{X}$ is finite type over a field, then (1) and (2) are equivalent to: (3) $\mathcal{X} = [\mathrm{Spec} A/G]$ with G an affine group scheme finite type over k. The implication that quotient stacks have the resolution property was proven by Thomason. The second main theorem is that if $\mathcal{X}$ is a smooth Deligne-Mumford stack over a field which has a finite and generically trivial stabilizer group $I_{\mathcal{X}} \to \mathcal{X}$ and whose coarse moduli space is a scheme with affine diagonal, then $\mathcal{X}$ has the resolution property. Another cool result states that if $\mathcal{X}$ is a

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[2]In the literature, *quotient stack* often means a stack of the form $[X/G]$ with X an algebraic space and G a subgroup scheme of GL_n rather than an arbitrary flat group scheme.

noetherian algebraic stack satisfying the resolution property, then $\mathcal{X}$ has affine diagonal if and only if the closed points have affine stabilizer.

- Kresch: *On the Geometry of Deligne-Mumford Stacks* [Kre09]
  This article summarizes general structure results of Deligne-Mumford stacks (of finite type over a field) and contains some interesting results concerning quotient stacks. It is shown that any smooth, separated, generically tame Deligne-Mumford stack with quasi-projective coarse moduli space is a quotient stack $[Y/G]$ with Y quasi-projective and G an algebraic group. If $\mathcal{X}$ is a Deligne-Mumford stack whose coarse moduli space is a scheme, then $\mathcal{X}$ is Zariski-locally a quotient stack if and only if it admits a Zariski-open covering by stack quotients of schemes by finite groups. If $\mathcal{X}$ is a Deligne-Mumford stack proper over a field of characteristic 0 with coarse moduli space Y, then: Y is projective and $\mathcal{X}$ is a quotient stack $\iff$ Y is projective and $\mathcal{X}$ possesses a generating sheaf $\iff$ $\mathcal{X}$ admits a closed embedding into a smooth proper DM stack with projective coarse moduli space. This motivates a definition that a Deligne-Mumford stack is *projective* if there exists a closed embedding into a smooth, proper Deligne-Mumford stack with projective coarse moduli space.

- Kresch, Vistoli *On coverings of Deligne-Mumford stacks and surjectivity of the Brauer map* [KV04]
  It is shown that over a field k and for a fixed n, the following two statements are equivalent: (1) every smooth separated generically tame Deligne-Mumford stack over k of dimension n with quasi-projective moduli space is a quotient stack and (2) the Azumaya Brauer group coincides with the cohomological Brauer group for smooth quasi-projective schemes over k of dimension n.

- Kresch: *Cycle Groups for Artin Stacks* [Kre99]
  It is shown that a reduced Artin stack finite type over a field with affine stabilizer groups admits a stratification by quotient stacks.

- Abramovich-Vistoli: *Compactifying the space of stable maps* [AV02]
  Lemma 2.2.3 establishes that for any separated Deligne-Mumford stack is étale-locally on the coarse moduli space a quotient stack $[U/G]$ where U affine and G a finite group. [Ols06b, Theorem 2.12] shows in this argument G is even the stabilizer group.

- Abramovich, Olsson, Vistoli: *Tame stacks in positive characteristic* [AOV08]
  This paper shows that a tame Artin stack is étale locally on the coarse moduli space a quotient stack of an affine by the stabilizer group.

- Alper: *On the local quotient structure of Artin stacks* [Alp09]
  It is conjectured that for an Artin stack $\mathcal{X}$ and a closed point $x \in \mathcal{X}$ with linearly reductive stabilizer, then there is an étale morphism $[V/G_x] \to \mathcal{X}$ with V an algebraic space. Some evidence for this conjecture is given. A simple deformation theory argument (based on ideas in [AOV08]) shows that it is true formally locally. A stack-theoretic proof of Luna's étale slice theorem is presented proving that for stacks $\mathcal{X} = [\mathrm{Spec} A/G]$ with G linearly reductive, then étale locally on the GIT quotient $\mathrm{Spec} A^G$, $\mathcal{X}$ is a quotient stack by the stabilizer.

5.5. Cohomology.

- Olsson: *Sheaves on Artin stacks* [Ols07b]

    This paper develops the theory of quasi-coherent and constructible sheaves proving basic cohomological properties. This paper corrects a mistake in [LMB00] in the functoriality of the lisse-étale site. The cotangent complex is constructed. In addition, the following theorems are proved: Grothendieck's Fundamental Theorem for proper morphisms, Grothendieck's Existence Theorem, Zariski's Connectness Theorem and finiteness theorem for proper pushoforwards of coherent and constructible sheaves.

- Behrend: *Derived l-adic categories for algebraic stacks* [Beh03]

    Proves the Lefschetz trace formula for algebraic stacks.

- Behrend: *Cohomology of stacks* [Beh04]

    Defines the de Rham cohomology for differentiable stacks and singular cohomology for topological stacks.

- Faltings: *Finiteness of coherent cohomology for proper fppf stacks* [Fal03]

    Proves coherence for direct images of coherent sheaves for proper morphisms.

- Abramovich, Corti, Vistoli: *Twisted bundles and admissible covers* [ACV03]

    The appendix contains the proper base change theorem for étale cohomology for tame Deligne-Mumford stacks.

5.6. Existence of finite covers by schemes. The existence of finite covers of Deligne-Mumford stacks by schemes is an important result. In intersection theory on Deligne-Mumford stacks, it is an essential ingredient in defining proper pushforward for non-representable morphisms. There are several results about $\overline{\mathcal{M}}_g$ relying on the existence of a finite cover by a *smooth* scheme which was proven by Looijenga. Perhaps the first result in this direction is [Ses72, Theorem 6.1] which treats the equivariant setting.

- Vistoli: *Intersection theory on algebraic stacks and on their moduli spaces* [Vis89]

    If $\mathcal{X}$ is a Deligne-Mumford stack with a moduli space (ie. a proper morphism which is bijective on geometric points), then there exists a finite morphism $X \to \mathcal{X}$ from a scheme X.

- Laumon, Moret-Bailly: [LMB00, Chapter 16]

    As an application of Zariski's main theorem, Theorem 16.6 establishes: if $\mathcal{X}$ is a Deligne-Mumford stack finite type over a noetherian scheme, then there exists a finite, surjective, generically étale morphism $Z \to \mathcal{X}$ with Z a scheme. It is also shown in Corollary 16.6.2 that any noetherian normal algebraic space is isomorphic to the algebraic space quotient X'/G for a finite group G acting a normal scheme X.

- Edidin, Hassett, Kresch, Vistoli: *Brauer Groups and Quotient stacks* [EHKV01]

    Theorem 2.7 states: if $\mathcal{X}$ is an algebraic stack of finite type over a noetherian ground scheme S, then the diagonal $\mathcal{X} \to \mathcal{X} \times_S \mathcal{X}$ is quasi-finite if and only if there exists a finite surjective morphism $X \to F$ from a scheme X.

- Kresch, Vistoli: *On coverings of Deligne-Mumford stacks and surjectivity of the Brauer map* [KV04]

    It is proved here that any separated Deligne-Mumford quotient stack $\mathcal{X}$ finite type over a field with quasi-projective coarse moduli space admits

a finite, flat cover by a quasi-projective scheme. If $\mathcal{X}$ is smooth, then the cover can be chosen to be smooth.

- Olsson: *On proper coverings of Artin stacks* [Ols05]

    Proves that if $\mathcal{X}$ is an Artin stack separated and finite type over S, then there exists a proper surjective morphism $X \to \mathcal{X}$ from a scheme X quasi-projective over S. As an application, Olsson proves coherence and constructibility of direct image sheaves under proper morphisms. As an application, he proves Grothendieck's existence theorem for proper Artin stacks.

5.7. Rigidification. Rigidification is a process for removing a flat subgroup from the inertia. For example, if X is a projective variety, the morphism from the Picard stack to the Picard scheme is a rigidification of the group of automorphism $\mathbb{G}_m$.

- Abramovich, Corti, Vistoli: *Twisted bundles and admissible covers* [ACV03]

    Let $\mathcal{X}$ be an algebraic stack over S and H be a flat, finitely presented separated group scheme over S. Assume that for every object $\xi \in \mathcal{X}(T)$ there is an embedding $H(S) \hookrightarrow \mathrm{Aut}_{\mathcal{X}(T)}(\xi)$ which is compatible under pullbacks in the sense that for every arrow $\phi : \xi \to \xi'$ over $f : T \to T'$ and $g \in H(T)$, $g \circ \phi = \phi \circ f^* g$. Then there exists an algebraic stack $\mathcal{X} /\!\!/ H$ and a morphism $\rho : \mathcal{X} \to \mathcal{X} /\!\!/ H$ which is an fppf gerbe such that for every $\xi \in \mathcal{X}(T)$, the morphism $\mathrm{Aut}_{\mathcal{X}(T)}(\xi) \to \mathrm{Aut}_{\mathcal{X} /\!\!/ H(T)}(\xi)$ is surjective with kernel $H(T)$.

- Romagny: *Group actions on stacks and applications* [Rom05]

    Discusses how group actions behave with respect to rigidifications.

- Abramovich, Graber, Visolit: *Gromov-Witten theory for Deligne-Mumford stacks* [AGV08]

    The appendix gives a summary of rigidification as in [ACV03] with two alternative interpretations. This paper also contains constructions for gluing algebraic stacks along closed substacks and for taking roots of line bundles.

- Abramovich, Olsson, Vistoli: *Tame stacks in positive characteristic* ([AOV08])

    The appendix handles the more complicated situation where the flat subgroup stack of the inertia $H \subseteq I_{\mathcal{X}}$ is normal but not necessarily central.

5.8. Stacky curves.

- Abramovich, Vistoli: *Compactifying the space of stable maps* [AV02]

    This paper introduces *twisted curves*. If one defines the moduli space of stable maps into a stacks using stable curves, the result is not compact. By using maps from twisted curves, the authors define a proper moduli stack.

- Behrend, Noohi: *Uniformization of Deligne-Mumford curves* [BN06]

    Proves a uniformization theorem of Deligne-Mumford analytic curves.

5.9. Hilbert, Quot, Hom and branchvariety stacks.

- Vistoli: *The Hilbert stack and the theory of moduli of families* [Vis91]

    If $\mathcal{X}$ is a algebraic stack separated and locally of finite type over a locally noetherian and locally separated algebraic space S, Vistoli defines the Hilbert stack $\mathcal{H}\mathrm{ilb}(\mathcal{F}/S)$ parameterizing finite and unramifed morphisms

from proper schemes. It is claimed without proof that $\mathcal{H}\text{ilb}(\mathcal{F}/S)$ is an algebraic stack. As a consequence, it is proved that with $\mathcal{X}$ as above, the Hom stack $\mathcal{H}\text{om}_S(T, \mathcal{X})$ is an algebraic stack if T is proper and flat over S.

- Olsson, Starr: *Quot functors for Deligne-Mumford stacks* [OS03]
  If $\mathcal{X}$ is a Deligne-Mumford stack separated and locally of finite presentation over an algebraic space S and $\mathcal{F}$ is a locally finitely-presented $\mathcal{O}_{\mathcal{X}}$-module, the quot functor $\text{Quot}(\mathcal{F}/\mathcal{X}/S)$ is represented by an algebraic space separated and locally of finite presentation over S. This paper also defines generating sheaves and proves existence of a generating sheaf for tame, separated Deligne-Mumford stacks which are global quotient stacks of a scheme by a finite group.

- Olsson: *Hom-stacks and Restrictions of Scalars* [Ols06b]
  Suppose $\mathcal{X}$ and $\mathcal{Y}$ are Artin stacks locally of finite presentation over an algebraic space S with finite diagonal with $\mathcal{X}$ proper and flat over S such that fppf-locally on S, $\mathcal{X}$ admits a finite finitely presented flat cover by an algebraic space. Then $\text{Hom}_S(\mathcal{X}, \mathcal{Y})$ is an Artin stack locally of finite presentation over S.

- Alexeev and Knutson: Complete moduli spaces of branchvarieties ([AK06])
  They define a branchvariety of $\mathbb{P}^n$ as a finite morphism $X \to \mathbb{P}^n$ from a *reduced* scheme X. They prove that the moduli stack of branchvarieties with fixed Hilbert polynomial and total degrees of i-dimensional components is a proper Artin stack with finite stabilizer. They compare the stack of branchvarieties with the Hilbert scheme, Chow scheme and moduli space of stable maps.

- Lieblich: *Remarks on the stack of coherent algebras* [Lie06]
  This paper constructs a generalization of Alexeev and Knutson's stack of branch-varieties over a scheme Y by building the stack as a stack of algebras over the structure sheaf of Y. Existence proofs of Quot and Hom spaces are given.

- Starr: *Artin's axioms, composition, and moduli spaces* [Sta06]
  As an application of the main result, a common generalization of Vistoli's Hilbert stack [Vis91] and Alexeev and Knutson's stack of branchvarieties [AK06] is provided. If $\mathcal{X}$ is an algebraic stack locally of finite type over an an excellent scheme S with finite diagonal, then the stack $\mathcal{H}$ parameterizing morphisms $g : T \to \mathcal{X}$ from a proper algebraic space T with a G-ample line bundle L is an Artin stack locally of finite type over S.

- Lundkvist and Skjelnes: *Non-effective deformations of Grothendiecks Hilbert functor* [LS08]
  Shows that the Hilbert functor of a non-separated scheme is not represented since there are non-effective deformations.

5.10. Toric stacks. Toric stacks provide a great class of examples and a natural testing ground for conjectures due to the dictionary between the geometry of a toric stack and the combinatorics of its stacky fan in a similar way that toric varieties provide examples and counterexamples in scheme theory.

- Borisov, Chen and Smith: *The orbifold Chow ring of toric Deligne-Mumford stacks* [BCS05]

    Inspired by Cox's construction for toric varieties, this paper defines smooth toric DM stacks as explicit quotient stacks associated to a combinatorial object called a *stacky fan.*

- Iwanari: *The category of toric stacks* [Iwa09]

    This paper defines a *toric triple* as a smooth Deligne-Mumford stack $\mathcal{X}$ with an open immersion $\mathbb{G}_m \hookrightarrow \mathcal{X}$ with dense image (and therefore $\mathcal{X}$ is an orbifold) and an action $\mathcal{X} \times \mathbb{G}_m \to \mathcal{X}$. It is shown that there is an equivalence between the 2-category of toric triples and the 1-category of stacky fans. The relationship between toric triples and the definition of smooth toric DM stacks in [BCS05] is discussed.

- Iwanari: *Integral Chow rings for toric stacks* [Iwa07]

    Generalizes Cox's Δ-collections for toric varieties to toric orbifolds.

- Perroni: *A note on toric Deligne-Mumford stacks* [Per08]

    Generalizes Cox's Δ-collections and Iwanari's paper [Iwa07] to general smooth toric DM stacks.

- Fantechi, Mann, and Nironi: *Smooth toric DM stacks* [BF07]

    This paper defines a smooth toric DM stack as a smooth DM stack $\mathcal{X}$ with the action of a DM torus $\mathcal{T}$ (ie. a Picard stack isomorphic to $T \times BG$ with G finite) having an open dense orbit isomorphic to $\mathcal{T}$. They give a "bottom-up description" and prove an equivalence between smooth toric DM stacks and stacky fans.

5.11. Theorem on formal functions and Grothendieck's Existence Theorem. These papers give generalizations of the theorem on formal functions [EGA, III.4.1.5] (sometimes referred to Grothendieck's Fundamental Theorem for proper morphisms) and Grothendieck's Existence Theorem [EGA, III.5.1.4].

- Knutson: *Algebraic spaces* [Knu71, Chapter V]

    Generalizes these theorems to algebraic spaces.

- Abramovich-Vistoli: *Compactifying the space of stable maps* [AV02, A.1.1]

    Generalizes these theorems to tame Deligne-Mumford stacks

- Olsson and Starr: *Quot functors for Deligne-Mumford stacks* [OS03]

    Generalizes these theorems to separated Deligne-Mumford stacks.

- Olsson: *On proper coverings of Artin stacks* [Ols05]

    Provides a generalization to proper Artin stacks.

- Conrad: *Formal GAGA on Artin stacks* [Con]

    Provides a generalization to proper Artin stacks and proves a formal GAGA theorem.

- Olsson: *Sheaves on Artin stacks* [Ols07b]

    Provides another proof for the generalization to proper Artin stacks.

5.12. Group actions on stacks. Actions of groups on algebraic stacks naturally appear. For instance, symmetric group S_n acts on $\overline{\mathcal{M}}_{g,n}$ and for an action of a group G on a scheme X, the normalizer of G in $\mathrm{Aut}(X)$ acts on $[X/G]$. Furthermore, torus actions on stacks often appear in Gromov-Witten theory.

- Romagny: *Group actions on stacks and applications* [Rom05]

This paper makes precise what it means for a group to act on an algebraic stack and proves existence of fixed points as well as existence of quotients for actions of group schemes on algebraic stacks. See also Romagny's earlier note [Rom03].

5.13. Taking roots of line bundles. This useful construction was discovered independently by Cadman and by Abramovich, Graber and Vistoli. Given a scheme X with an effective Cartier divisor D, the rth root stack is an Artin stack branched over X at D with a μ_r stabilizer over D and scheme-like away from D.

- Charles Cadman *Using Stacks to Impose Tangency Conditions on Curves* [Cad07]
- Abramovich, Graber, Vistoli: *Gromov-Witten theory for Deligne-Mumford stacks* [AGV08]

5.14. Other papers.

- Lieblich: *Moduli of twisted sheaves* [Lie07]
  This paper contains a summary of gerbes and twisted sheaves. If $\mathcal{X} \to X$ is a μ_n-gerbe with X a projective relative surface with smooth connected geometric fibers, it is shown that the stack of semistable $\mathcal{X}$-twisted sheaves is an Artin stack locally of finite presentation over S. This paper also develops the theory of associated points and purity of sheaves on Artin stacks.

- Lieblich, Osserman: *Functorial reconstruction theorem for stacks* [LO08]
  Proves some surprising and interesting results on when an algebraic stack can be reconstructed from its associated functor.

- David Rydh: *Noetherian approximation of algebraic spaces and stacks* [Ryd09]
  This paper shows that every quasi-compact algebraic stack with quasi-finite diagonal can be approximated by a noetherian stack. There are applications to removing the noetherian hypothesis in results of Chevalley, Serre, Zariski and Chow.

6. Stacks in other fields

- Behrend and Noohi: *Uniformization of Deligne-Mumford curves* [BN06]
  Gives an overview and comparison of topological, analytic and algebraic stacks.

- Behrang Noohi: *Foundations of topological stacks I* [Noo05]
- David Metzler: *Topological and smooth stacks* [Met05]

7. Higher stacks

- Lurie: *Higher topos theory* [Lur09f]
- Lurie: *Derived Algebraic Geometry I - V* [Lur09a], [Lur09b], [Lur09c], [Lur09d], [Lur09e]
- Toën: *Higher and derived stacks: a global overview* [Toë09]
- Toën and Vezzosi: *Homotopical algebraic geometry I, II* [TV05], [TV08]

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