

# THE FIRST 200,000,001 ZEROS OF RIEMANN'S ZETA FUNCTION

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## ABSTRACT

We describe extensive computations which show that Riemann's zeta function  $\zeta(s)$  has exactly 200,000,001 zeros of the form  $\sigma + it$  in the region  $0 < t < 81,702,130.19$ ; all these zeros are simple and lie on the line  $\sigma = 1/2$ . This extends a result for the first 81,000,001 zeros, established by Brent in [1]. Counts of the numbers of Gram blocks of various types and the failures of "Rosser's rule" are given.

## COMMENTS

Only the Abstract is given here. The full paper, which appeared as [3], extended the results of [1]. A revision appeared as [2]. For further work, see [4].

## REFERENCES

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- [4] J. van de Lune, H. J. J. te Riele and D. T. Winter, "On the zeros of the Riemann zeta function in the critical strip, IV", *Mathematics of Computation* 46, 1986, 667–681.

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