Photon Spheres around Higher Dimensional Black Holes

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Overview

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   - General relativity
   - Photon spheres
2. Photon spheres in Kerr
   - Kerr black holes
   - Geodesic equations and selected orbits
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   - Myers Perry black holes
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Spacetime in General Relativity

In general relativity, the shape of spacetime is described by a manifold, together with a Lorentzian metric $g_{\mu\nu}$ satisfying the Einstein Field Equations:

Einstein Field Equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}$$

In four dimensions, these are a set of 16 coupled hyperbolic-elliptic non-linear partial differential equations.

The motion of particles

The motion of particles in a fixed spacetime background is determined by the geodesic equations:

Geodesic equations

$$\frac{dx^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0$$

The motion of light is described by a *null geodesic*. That is, a solution to the geodesic equations which also satisfies $g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = 0$

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Photon spheres

Given a particular spacetime background, we can study the movement of particles (or light) moving in this background using the geodesic equations.

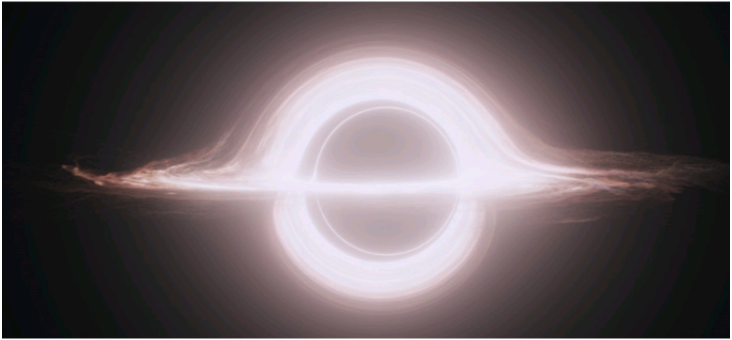


Figure: Created by Double Negative team using DNGR, and TM & ©Warner Bros. Entertainment Inc.

I'm interested in null geodesics around black hole spacetimes. In particular, I'm interested in null geodesics which have a constant radius.

These geodesics correspond to light orbiting around a black hole.

The simplest black hole

The simplest example of a black hole is the Schwarzchild black hole. This is the (unique!) spherically symmetric, static vacuum solution to the Einstein Field Equations in 4 dimensions.

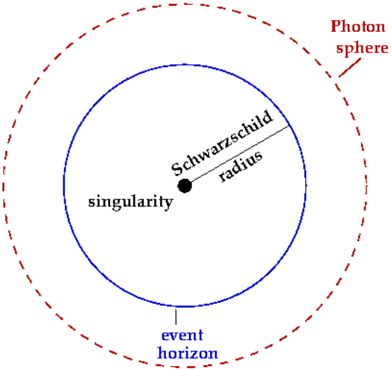
The metric is relatively simple:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{\left(1 - \frac{2M}{r}\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

The photon sphere in Schwarzschild

The Schwarzschild metric describes the spacetime around a spherically symmetric black hole. The metric has an event horizon, located at $r = 2M$.

Null geodesics with constant radius exist only at $r = 3M$. The collection of all orbits form the *photon sphere* of the black hole.



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The Kerr spacetime

The Kerr metric describes the spacetime outside a rotating, axisymmetric black hole. The metric is:

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - 2\left(\frac{2Mr}{\Sigma}\right) a \sin^2\theta dt d\phi + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\mathcal{A}}{\Sigma}\sin^2\theta d\phi^2$$

where

$$\begin{aligned}
\Sigma &= r^2 + a^2\cos^2\theta \\
\Delta &= r^2 + a^2 - 2Mr \\
\mathcal{A} &= (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta
\end{aligned}$$

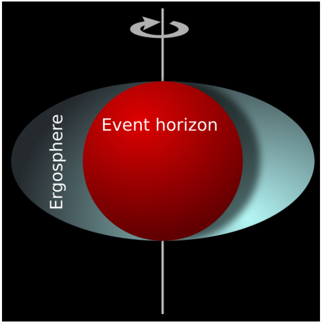
and a is the rotation parameter of the black hole.

Frame dragging

The Kerr metric exhibits unusual properties, such as frame dragging.

Frame dragging causes objects close to the black hole to corotate with the black hole, even if they were initially counterrotating.

Spacetime itself is dragged around the black hole.



This frame dragging changes the properties of photon orbits and makes them much more interesting.

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Geodesic equations for null geodesics in Kerr

The geodesic equations can be written as the following set of coupled ODEs:

$$
\begin{aligned}
\Delta\Sigma\dot{t} &= \mathcal{A}E - 2MraL \\
\Sigma^2\dot{r}^2 &= E^2r^4 - \left(a^2E^2 - L^2 - \mathcal{Q}\right)r^2 + 2M\left[(aE - L)^2 + \mathcal{Q}\right]r - a^2\mathcal{Q} \\
\Sigma^2\dot{\theta}^2 &= \mathcal{Q} - \left[\frac{L^2}{\sin^2\theta} - E^2a^2\right]\cos^2\theta \\
\Delta\Sigma\dot{\phi} &= 2MraE + (\Sigma - 2Mr)\frac{L}{\sin^2\theta}
\end{aligned}
$$

An analysis of the geodesic equations allows one to determine properties of spherical orbits. The physically allowed orbits are parametrised by a single parameter, $\Phi = L/E$.

This parameter determines the radius of the orbit, as well as the maximum *latitude* of the orbit.

The geodesic equations can be solved numerically to obtain explicit examples of such orbits.

Selected orbits 1: Non planar

One of the interesting features of these orbits is the possibility of non-planar orbits.

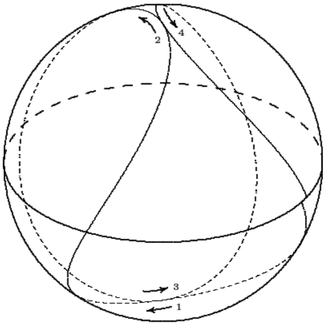

Figure: Two latitudinal oscillations for an orbit with $\Phi = 0$

Selected orbits 2: Frame Dragging

This orbit does not have a fixed azimuthal direction. At the equator, the photon is corotating, but towards the poles it is counter rotating.

This is attributed to the Lense-Thirring effect: Frame dragging is strongest near the equator.

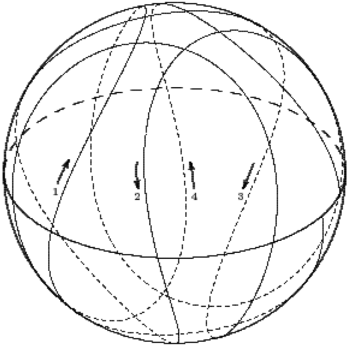

Figure: Two latitudinal oscillations for an orbit with $\Phi = -M$

Selected orbits 3: Maximum latitude

Generically, the orbits will have a maximum latitude they are able to obtain.

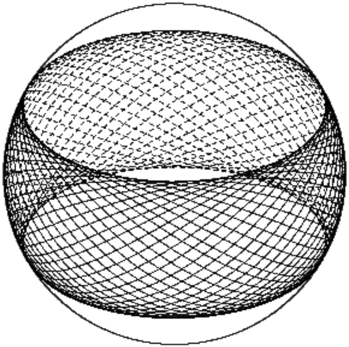

Figure: Two latitudinal oscillations for an orbit with $\Phi = -6M$

Overview

The Myers-Perry spacetime

The 5D Myers-Perry black hole we will look at is the simplest generalisation of the Kerr spacetime. It describes an axisymmetric, stationary spacetime in 5 dimensions with *one* plane of rotation. The metric is:

$$ds^2 = -\left(1 - \frac{\mu r}{\Sigma}\right) dt^2 - \frac{2\mu r}{\Sigma} a \sin^2\theta dt d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{\mu a^2 \sin^2\theta}{\Sigma}\right) \sin^2\theta d\phi^2 + r^2 \cos^2\theta d\psi^2$$

In general, a Myers-Perry black hole describes a stationary black hole in d dimensions rotating in $N \equiv \lfloor \frac{d-1}{2} \rfloor$ independent planes.

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Geodesic equations for null geodesics in 5D MP

The geodesic equations can be written as the following set of coupled ODEs:

$$
\begin{aligned}
\Delta\Sigma\dot{t} &= E\Sigma(r^2+a^2) + E\mu\sin^2\theta - \mu a L \\
\Sigma^2\dot{r}^2 &= E^2r^4 + (a^2E^2 - L^2 - \mathcal{Q})r^2 + 2M[(L-aE)^2 + \mathcal{Q}]r \\
&\quad - a^2(\mathcal{Q}+N^2) + \frac{2MN^2a^2}{r} - \frac{N^2a^4}{r^2} \\
\Sigma^2\dot{\theta}^2 &= \mathcal{Q} - \left(\frac{L^2}{\sin^2\theta} - a^2E^2\right)\cos^2\theta - \frac{N^2}{\cos^2\theta} \\
\Delta\Sigma\dot{\phi} &= \mu a E + (\Sigma-\mu)\frac{L\Delta}{\sin^2\theta} \\
\dot{\psi} &= \frac{N}{r^2\cos^2\theta}
\end{aligned}
$$

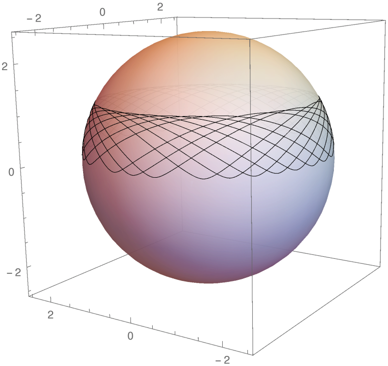
Again, an analysis of these geodesic equations can be done to determine the allowed range of parameters for physical orbits.

The orbits now depend on two parameters: The usual parameter Φ of the photon, as well as a new parameter Ψ, which is related to the angular momentum in the 'new' direction.

Selected Orbits 1: Bands away from the equator and poles

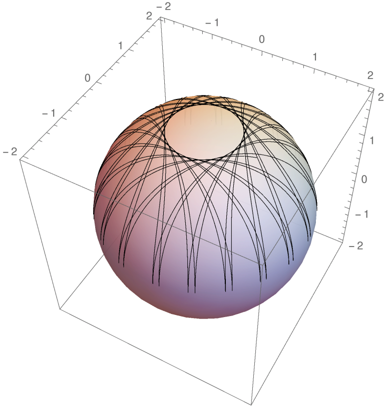
The first interesting thing to note about these orbits is that they are generically confined to bands which exclude both the equator and the poles.

Like in the case of Kerr, we get quasiperiodic behaviour.



Selected Orbits 2: Cusps?

The appearance of cusps seems bizarre, but is really just an artifact of the projection onto three spatial dimensions.



Where do the cusps come from?

It's easiest to see how we get cusps if we plot the path of the geodesic through (θ, ϕ, ψ) space.

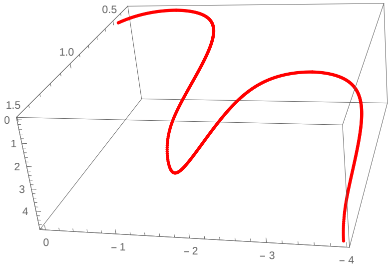


Figure: A plot of the previous geodesic, through (θ, ϕ, ψ) space

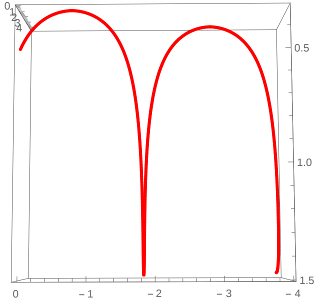


Figure: The same geodesic seen from above looks like it has cusps.

Overview

Further work

In addition to finishing the characterisation of orbits, it would be interesting to examine bound null geodesics in the following spacetimes:

- 5D Myers-Perry black holes with *two* planes of rotation
- Arbitrary dimension Myers-Perry black holes
- Black ring?
- Black Saturn?
- ???

Thanks for listening!

References

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