Algebra 1 Honours, ASE add-on, Assignment 1
Due Friday August 25.

(1) Let H be a group, P a set, and $\phi : H \to P$ a bijection. Show that there is a unique group structure on P such that ϕ is a group homomorphism (and so an isomorphism). Apply this to the bijection $G \to P$, where G is the Rubik's group and P is the set of all positions of the cube to put a group structure on the set of all positions of the cube. Writing positions as quadruples $(\rho, \sigma, \boldsymbol{x}, \boldsymbol{y})$ as in Theorem 8 of the lecture notes, what is the product of two quadruples? (This can used to great effect in the rest of the exercises.)

(2) Define a group G_2 to be the analogue of the Rubik's group for the $2 \times 2 \times 2$ cube, and show there is a surjective group homomorphism $G \to G_2$. Describe the kernel using the quadruple description of group elements. In particular, what is the order of the kernel? Find a specific move that is in the kernel.

(3) A *two squares subgroup* of G is a subgroup generated by the squares of two face rotations. Find the structure of the subgroups $\langle F^2, B^2 \rangle$ and $\langle F^2, R^2 \rangle$ (by find the structure I mean find all of the elements, and determine what their products are). Show these two subgroups are not isomorphic, and that any two squares subgroup is isomorphic to one of the above.

(4) Find the center of the Rubik's group G.