

Algebra 1 Honours, ASE add-on, Assignment 2
Due Friday September 22.

There are two options for this homework.

Option 1: Do Question 1 and any two of Questions 2-5.

- (1) Show how using $[D', R']$ in the last step of the solution from lecture can reduce the total number of moves.
- (2) Show that no corners can be twisted or edges flipped in the Two Squares group $\langle U^2, D^2, L^2, R^2, F^2, B^2 \rangle$.
- (3) Find a move that twists or flips every cubie, while leaving it in its start cubie. (Hint: the last question on Assignment 1 could help.)
- (4) Show the set of moves leaving the bottom layer fixed is a subgroup of G . Find a set of generators of this subgroup (and show they generate).
- (5) What is the index of $\langle F^2, R^2 \rangle$ in $\langle F, R \rangle$? The index of $\langle F^2 R^2 \rangle$ in $\langle F^2, R^2 \rangle$?

Option 2: This is more open ended. I'd like you to learn and explain some mathematical aspect of the cube that we didn't cover in lecture. Some possibilities are below. If you do this, and have questions about what is appropriate, please email me and we can discuss. Whatever you do, make sure it's nontrivial in some way.

- Define and analyze the “extended Rubik’s group” or “Rubik’s supergroup” that involves keeping track of the orientation of the center cubies.
- Generalize something(s) we did in class to the 4x4x4 case, or to the arbitrary $n \times n \times n$ case. A big question: is there a main theorem of cubology for these larger cubes? Even formulating such a theorem (without proving) seems like it would be non-trivial.
- Write a simple text program that inputs a move, e.g., LRU2B', and outputs the move in cycle notation, and/or in the quadruple notation we used in the main theorem and on the last assignment. I would be very happy if someone did this, and several of you could work together on this, if you're interested.
- Analyze how adding some additional algorithms could speed up the solution we discussed in lecture.
- Try to find ways of illustrating moves that helps a reader (and perhaps yourself) understand what's going on. For instance, why does the commutator $[L', URU']$ give a corner 3-cycle?
- Research “God’s number” for the cube. E.g., give a proof of a lower and upper bound, and try to understand something about the recent (computational) proof that God’s number is 20.