Chapter 9 Exercises

Data Analysis & Graphics Using R – Solutions to Exercises (April 24, 2004)

Preliminaries

```r
> library(nlme)
> library(DAAG)
```

The final two sentences of Exercise 1 are challenging! Exercises 1 & 2 should be asterisked.

Exercise 1

Repeat the calculations of Subsection 9.3.5, but omitting results from two vines at random. Here is code that will handle the calculation:

```r
n.omit <- 2
take <- rep(T, 48)
take[sample(1:48, n.omit)] <- F
kiwishade.lme <- lme(yield ~ shade, random = ~ 1 | block/plot,
data = kiwishade, subset = take)
VarCorr(kiwishade.lme)[4, 1] # Plot component of variance
VarCorr(kiwishade.lme)[5, 1] # Vine component of variance
```

Repeat this calculation five times, for each of n.omit = 2, 4, 6, 8, 10, 12 and 14. Plot (i) the plot component of variance and (ii) the vine component of variance, against number of points omitted. Based on these results, for what value of n.omit does the loss of vines begin to compromise results? Which of the two components of variance estimates is more damaged by the loss of observations? Comment on why this is to be expected.

For convenience, we place the central part of the calculation in a function. On slow machines, the code may take a minute or two to run.

```r
> trashvine <- function(n.omit = 2) {
+   k <- k + 1
+   n[k] <- n.omit
+   take <- rep(T, 48)
+   take[sample(1:48, n.omit)] <- F
+   kiwishade$take <- take
+   kiwishade.lme <- lme(yield ~ shade, random = ~ 1 | block/plot,
+                         data = kiwishade, subset = take)
+   varp <- as.numeric(VarCorr(kiwishade.lme)[4, 1])
+   varv <- as.numeric(VarCorr(kiwishade.lme)[5, 1])
+   c(varp, varv)
+ }
> varp <- numeric(35)
> varv <- numeric(35)
> n <- numeric(35)
> k <- 0
> data(kiwishade)
> for (n.omit in c(2, 4, 6, 8, 10, 12, 14)) for (i in 1:5) {
+   k <- k + 1
+   vec2 <- trashvine(n.omit = n.omit)
+   n[k] <- n.omit
```
As the number of vines that are omitted increases, the variance estimates can be
expected to show greater variability. The fraction omitted may not be large enough for
the effect to show clearly. Increasing the number of repeats for each value of n.omit
would help. The effect should be most evident on the between plot variance. Inaccuracy
in estimates of the between plot variance arise both from inaccuracy in the within plot
sums of squares and from loss of information at the between plot level.

At best it is possible only to give an approximate d.f. for the between plot estimate
of variance (some plots lose more vines than others), which complicates any evaluation
that relies on degree of freedom considerations.

Exercise 2
Repeat the previous exercise, but now omitting 1, 2, 3, 4 complete plots at random.

> trashplot <- function(n.omit = 2) {
+   k <- k + 1
+   n[k] <- n.omit
+   plotlev <- levels(kiwishade$plot)
+   use.lev <- sample(plotlev, length(plotlev) - n.omit)
+   kiwishade$take <- kiwishade$plot %in% use.lev
+   kiwishade.lme <- lme(yield ~ shade, random = ~1 | block/plot,
+     data = kiwishade, subset = take)
+   varp <- as.numeric(VarCorr(kiwishade.lme)[4, 1])
+   varv <- as.numeric(VarCorr(kiwishade.lme)[5, 1])
+   c(varp, varv)
+ }
> varp <- numeric(20)
> varv <- numeric(20)
> n <- numeric(20)
> k <- 0
> for (n.omit in 1:4) for (i in 1:5) {
+   k <- k + 1
+   vec2 <- trashplot(n.omit = n.omit)
+   n[k] <- n.omit
+   varp[k] <- vec2[1]
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+ varv[k] <- vec2[2]
+ }

Again, we plot the results:

Figure 2: Within, and between plots variance estimates, as functions of the number of whole plots (each consisting of four vines) that were omitted at random.

Omission of a whole plot loses 3 d.f. out of 36 for estimation of within plot effects, and 1 degree of freedom out of 11 for the estimation of between plot effects, i.e., a slightly greater relative loss. The effect on precision will be most obvious where the d.f. are already smallest, i.e., for the between plot variance. The loss of information on complete plots is inherently for serious, for the estimation of the between plot variance, than the loss of partial information (albeit on a greater number of plots) as will often happen in Exercise 1.

Exercise 3
The final sentence has been modified; see the list of Corrections

A time series of length 100 is obtained from an AR(1) model with $\sigma = 1$ and $\alpha = -.5$. What is the standard error of the mean? If the usual $\sigma / \sqrt{n}$ formula were used in constructing a confidence interval for the mean, with $\sigma$ defined as in Section 9.5.3, would it be too narrow or too wide?

If we know $\sigma$, then the usual $\sigma / \sqrt{n}$ formula will give an error that is too narrow; refer back to Subsection 9.5.3 on page 244.

The need to estimate $\sigma$ raises an additional complication. If $\sigma$ is estimated by fitting a time series model, e.g., using the function \texttt{ar()}, this estimate of $\sigma$ can be plugged into the formula in Subsection 9.5.3. The note that now follows covers the case where $\sigma^2$ is estimated using the formula

$$
\hat{\sigma}^2 = \frac{\sum(X_i - \bar{X})^2}{n - 1}
$$

The relevant theoretical results are not given in the text. Their derivation requires a knowledge of the algebra of expectations.

**Note 1:** We use the result (proved below)

$$
E[(X_i - \mu)^2] = \sigma^2/(1 - \alpha^2)
$$

and that

$$
E[\sum(X_i - \bar{X})^2] = \frac{1}{1 - \alpha^2}(n - 1 - \alpha)\sigma^2 \simeq \frac{1}{1 - \alpha^2}(n - 1)\sigma^2
$$

Hence, if the variance is estimated from the usual formula $\hat{\sigma}^2 = \frac{\sum(X_i - \bar{X})^2}{n - 1}$, the standard error of the mean will be too small by a factor of approximately $\sqrt{\frac{1 - \alpha}{1 + \alpha}}$. 


Note 2: We square both sides of
\[ X_t - \mu = \alpha(X_{t-1} - \mu) + \varepsilon_t \]
and take expectations. We have that
\[ E[(X_t - \mu)^2] = (1 - \alpha^2)E[(X_t - \mu)^2] + \sigma^2 \]
from which the result (eq.1) follows immediately. To derive \( E[\sum (X_i - \bar{X})^2] \), observe that
\[ E[\sum (X_i - \bar{X})^2] = E[(X_t - \mu)^2] - n(\bar{X} - \mu)^2 \]

Exercise 4
Use the ar function to fit the second order autoregressive model to the Lake Huron time series.

```r
> if (!exists("LakeHuron")) data(LakeHuron)
> ar(LakeHuron, order.max = 2)

Call:
ar(x = LakeHuron, order.max = 2)

Coefficients:
1 2
 1.054 -0.267

Order selected 2  sigma^2 estimated as 0.508

It might however be better not to specify the order, instead allowing the ar() function to choose it, based on the AIC criterion. For this to be valid, it is best to specify also method="mle". Fitting by maximum likelihood can for long series be very slow. It works well in this instance.

> ar(LakeHuron, method = "mle")

Call:
ar(x = LakeHuron, method = "mle")

Coefficients:
1 2
 1.044 -0.250

Order selected 2  sigma^2 estimated as 0.479

The AIC criterion chooses the order equal to 2.

Exercise 5
The data set Gun (nlme package) reports on the numbers of rounds fired per minute, by each of nine teams of gunners, each tested twice using each of two methods. In the nine teams, three were made of men with slight build, three with average, and three with heavy build. Is there a detectable difference, in number of rounds fired, between build type or between firing methods? For improving the precision of results, which would be better – to double the number of teams, or to double the number of occasions (from 2 to 4) on which each team tests each method?
It probably does not make much sense to look for overall differences in Method; this depends on Physique. We therefore nest Method within Physique.

```r
> if (!exists("Gun")) data(Gun)
> Gun.lme <- lme(rounds ~ Physique/Method, random = ~1 | Team,
+ data = Gun)
> summary(Gun.lme)
```

Linear mixed-effects model fit by REML
Data: Gun

AIC BIC logLik
143.0 154.2 -63.48

Random effects:
Formula: ~1 | Team
(Intercept) Residual
StdDev: 1.044 1.476

Fixed effects: rounds ~ Physique/Method

```
Value Std.Error DF t-value p-value
(Intercept) 23.589 0.4922 24 47.92 0.0000
Physique.L -0.966 0.8526 6 -1.13 0.3003
Physique.Q 0.191 0.8526 6 0.22 0.8306
PhysiqueSlight:MethodM2 -8.450 0.8524 24 -9.91 0.0000
PhysiqueAverage:MethodM2 -8.100 0.8524 24 -9.50 0.0000
PhysiqueHeavy:MethodM2 -8.983 0.8524 24 -10.54 0.0000
```

Correlation:

```
Physique.L 0.000
Physique.Q 0.000 0.000
PhysiqueSlight:MethodM2 -0.289 0.353 -0.204
PhysiqueAverage:MethodM2 -0.289 0.000 0.408 0.000
PhysiqueHeavy:MethodM2 -0.289 -0.353 -0.204 0.000 0.000
```

Standardized Within-Group Residuals:

```
Min Q1 Med Q3 Max
-2.15598 -0.64718 0.09983 0.63379 1.67448
```

Number of Observations: 36
Number of Groups: 9

A good way to proceed is to determine the fitted values, and present these in an interaction plot:

```r
> Gun.hat <- predict(Gun.lme)
> interaction.plot(Gun$Physique, Gun$Method, Gun.hat)
```

Differences between methods, for each of the three physiques, are strongly attested. These can be estimated within teams, allowing 24 degrees of freedom for each of these comparisons.

Clear patterns of change with Physique seem apparent in the plot. There are however too few degrees of freedom for this effect to appear statistically significant. Note however that the parameters that are given are for the lowest level of Method, i.e., for M1. Making M2 the baseline shows the effect as closer to the conventional 5% significance level.
The component of variance at the between teams level is of the same order of magnitude as the within teams component. Its contribution to the variance of team means (1.044^2) is much greater than the contribution of the within team component (1.476^2/4; there are 4 results per team). If comparison between physiques is the concern; it will be much more effective to double the number of teams; compare (1.044^2+1.476^2/4)/2 (=0.82) with 1.044^2+1.476^2/8 (=1.36).

Exercise 6
*The data set ergoStool (nlme package) has data on the amount of effort needed to get up from a stool, for each of nine individuals who each tried four different types of stool. Analyse the data both using aov() and using lme(), and reconcile the two sets of output. Was there any clear winner among the types of stool, if the aim is to keep effort to a minimum?*

For analysis of variance, specify

```r
> if (!exists("ergoStool")) data(ergoStool)
> aov(effort ~ Type + Error(Subject), data = ergoStool)
```

Call:
`aov(formula = effort ~ Type + Error(Subject), data = ergoStool)`

Grand Mean: 10.25

Stratum 1: Subject

Terms:
```
  Residuals
  Sum of Squares  66.5
  Deg. of Freedom  8
```

Residual standard error: 2.883

Stratum 2: Within

Terms:
```
  Type Residuals
  Sum of Squares  81.19  29.06
  Deg. of Freedom  3   24
```

Residual standard error: 1.100

Estimated effects may be unbalanced

For testing the Type effect for statistical significance, refer (81.19/3)/(29.06/24) (=22.35) with the F.3.24 distribution. The effect is highly significant.

This is about as far as it is possible to go with analysis of variance calculations. When Error() is specified in the aov model, R has no mechanism for extracting estimates. (There are mildly torturous ways to extract the information, which will not be further discussed here.)

For use of lme, specify

```r
> summary(lme(effort ~ Type, random = ~1 / Subject, data = ergoStool))
```
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Linear mixed-effects model fit by REML
Data: ergoStool
   AIC  BIC logLik
133.1 141.9 -60.57

Random effects:
Formula: ~1 | Subject
   (Intercept) Residual
   StdDev: 1.332 1.100

Fixed effects: effort ~ Type
   Value Std.Error DF t-value p-value
(Intercept) 8.556 0.5760 24 14.853 0.0000
TypeT2 3.889 0.5187 24 7.498 0.0000
TypeT3 2.222 0.5187 24 4.284 0.0003
TypeT4 0.667 0.5187 24 1.285 0.2110

Correlation:
   (Intr) TypeT2 TypeT3
TypeT2 -0.45
TypeT3 -0.45 0.50
TypeT4 -0.45 0.50 0.50

Standardized Within-Group Residuals:
   Min Q1 Med Q3 Max
-1.80200 -0.64317 0.05783 0.70100 1.63142

Number of Observations: 36
Number of Groups: 9

Observe that $1.100295^2$ (Residual StdDev) is very nearly equal to $29.06/24$ obtained from the analysis of variance calculation.

Also the Stratum 1 mean square of $66.5/8 (=8.3125)$ from the analysis of variance output is very nearly equal to $1.3325^2 + 1.100295^2/4 (= 2.078)$ from the lme output.

Exercise 7
*In the data set MathAchieve (nlme package), the factors Minority (levels yes and no) and sex, and the variable SES (socio-economic status) are clearly fixed effects. Discuss how the decision whether to treat School as a fixed or as a random effect might depend on the purpose of the study? Carry out an analysis that treats School as a random effect. Are differences between schools greater than can be explained by within school variation?*

School should be treated as a random effect if the intention is to generalize results to other comparable schools. If the intention is to apply them to other pupils or classess within those same schools, it should be taken as a fixed effect.

For the analysis of these data, both SES and MEANSES should be included in the model. Then the coefficient of MEANSES will measure between school effects, while the coefficient of SES will measure within school effects.

```r
> if (!exists("MathAchieve")) data(MathAchieve)
> MathAch.lme <- lme(MathAch ~ Minority * Sex * (MEANSES + SES),
+  random = ~1 | School, data = MathAchieve)
> summary(MathAch.lme)
```
Linear mixed-effects model fit by REML
Data: MathAchieve

AIC  BIC  logLik
46344  46441  -23158

Random effects:
Formula: ~1 | School
(Intercept)  Residual
StdDev:  1.585  5.982

Fixed effects: MathAch ~ Minority * Sex * (MEANSES + SES)

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<th></th>
<th>Value</th>
<th>Std.Error</th>
<th>DF</th>
<th>t-value</th>
<th>p-value</th>
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Correlation:

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Standardized Within-Group Residuals:

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<th>Q3</th>
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</table>
Number of Observations: 7185  
Number of Groups: 160  
The between school component of variance (1.585^2) is 5.51, compared with a within school component that equals 35.79. To get a confidence intervals for the square roots of these variances, specify:

> intervals(MathAch.lme)

Approximate 95% confidence intervals

<table>
<thead>
<tr>
<th>Fixed effects:</th>
<th>lower</th>
<th>est.</th>
<th>upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>13.7112</td>
<td>14.0765</td>
<td>14.4417</td>
</tr>
<tr>
<td>MinorityYes</td>
<td>-3.6164</td>
<td>-3.0679</td>
<td>-2.5194</td>
</tr>
<tr>
<td>SexFemale</td>
<td>-1.6423</td>
<td>-1.2772</td>
<td>-0.9122</td>
</tr>
<tr>
<td>MEANSES</td>
<td>1.7817</td>
<td>2.8105</td>
<td>3.8394</td>
</tr>
<tr>
<td>SES</td>
<td>1.6233</td>
<td>1.9919</td>
<td>2.3604</td>
</tr>
<tr>
<td>MinorityYes:SexFemale</td>
<td>-0.2742</td>
<td>0.4623</td>
<td>1.1989</td>
</tr>
<tr>
<td>MinorityYes:MEANSES</td>
<td>-0.6320</td>
<td>0.7255</td>
<td>2.0830</td>
</tr>
<tr>
<td>MinorityYes:SES</td>
<td>-1.6649</td>
<td>-0.9904</td>
<td>-0.3160</td>
</tr>
<tr>
<td>SexFemale:MEANSES</td>
<td>-1.6992</td>
<td>-0.5740</td>
<td>0.5512</td>
</tr>
<tr>
<td>SexFemale:SES</td>
<td>-0.0015</td>
<td>0.5166</td>
<td>1.0348</td>
</tr>
<tr>
<td>MinorityYes:SexFemale:MEANSES</td>
<td>-1.0578</td>
<td>0.7132</td>
<td>2.4841</td>
</tr>
<tr>
<td>MinorityYes:SexFemale:SES</td>
<td>-1.0284</td>
<td>-0.1103</td>
<td>0.8078</td>
</tr>
</tbody>
</table>

attr(,"label")

[1] "Fixed effects:"]

Random Effects:

<table>
<thead>
<tr>
<th>Level: School</th>
<th>lower</th>
<th>est.</th>
<th>upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>sd((Intercept))</td>
<td>1.363</td>
<td>1.585</td>
<td>1.843</td>
</tr>
</tbody>
</table>

Within-group standard error:

<table>
<thead>
<tr>
<th>lower</th>
<th>est.</th>
<th>upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.884</td>
<td>5.982</td>
<td>6.082</td>
</tr>
</tbody>
</table>

The 95% confidence interval for the between school component of variance ranges from 1.36 to 1.84. The confidence interval excludes 0. Try also

> intervals(MathAch.lme, level = 0.9999)

Approximate 99.99% confidence intervals

<table>
<thead>
<tr>
<th>Fixed effects:</th>
<th>lower</th>
<th>est.</th>
<th>upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>13.3512</td>
<td>14.0765</td>
<td>14.8018</td>
</tr>
<tr>
<td>MinorityYes</td>
<td>-4.1571</td>
<td>-3.0679</td>
<td>-1.9786</td>
</tr>
<tr>
<td>SexFemale</td>
<td>-2.0022</td>
<td>-1.2772</td>
<td>-0.5523</td>
</tr>
<tr>
<td>MEANSES</td>
<td>0.7309</td>
<td>2.8105</td>
<td>4.8902</td>
</tr>
<tr>
<td>SES</td>
<td>1.2599</td>
<td>1.9919</td>
<td>2.7238</td>
</tr>
<tr>
<td>MinorityYes:SexFemale</td>
<td>-1.0003</td>
<td>0.4623</td>
<td>1.9250</td>
</tr>
<tr>
<td>MinorityYes:MEANSES</td>
<td>-1.9703</td>
<td>0.7255</td>
<td>3.4214</td>
</tr>
</tbody>
</table>
MinorityYes:SES -2.3299 -0.9904 0.3490  
SexFemale:MEANSES -2.8086 -0.5740 1.6606  
SexFemale:SES -0.5123 0.5166 1.5456  
MinorityYes:SexFemale:MEANSES -2.8037 0.7132 4.2300  
MinorityYes:SexFemale:SES -1.9335 -0.1103 1.7129

attr(,"label")

[1] "Fixed effects:")

Random Effects:
Level: School

sd((Intercept)) 1.175 1.585 2.139

Within-group standard error:
lower  est. upper
5.789 5.982 6.182

Zero is again excluded, still by a substantial margin.

The number of results for school varies between 14 and 67. Thus, the relative contribution to class means is 5.51 and a number that is at most 5.982429^2/14 = 2.56.

---

**Exercise 8**
The function `Box.test()` (in `ts`) may be used to compare the the straight line model with uncorrelated errors that was fitted in Section 9.6 against the alternative of autocorrelation at some lag greater than zero. Try, e.g.,

```r
Box.test(resid(lm(detrain ~ detSOI, data = detsoi)),
          type="Ljung-Box", lag=20)
```

It is necessary to guess at the highest possible lag at which an autocorrelation is likely. The number should not be too large; so that the flow-on effect from autocorrelation at lower lags is still evident. A common, albeit arbitrary choice, is a lag of 20, as here. Try running the test with `lag` set to values of 1 (the default), 15, 25 and 30. Comment on the different results.

The calculation for a lag of 20 was given on page 251. Here are the results for the other suggested lags:

```r
> if (!exists("bomsoi")) data(bomsoi)
> detsoi <- data.frame(detSOI = bomsoi[, "SOI"] - lowess(bomsoi[, + "SOI"]$y, detrain = log(bomsoi$avrain - 250) - lowess(log(bomsoi$avrain + 250))$y)
> row.names(detsoi) <- paste(1900:2001)
> Box.test(resid(lm(detrain ~ detSOI, data = detsoi)), type = "Ljung-Box",
          + lag = 15)
Box-Ljung test
data: resid(lm(detrain ~ detSOI, data = detsoi))
X-squared = 30.11, df = 15, p-value = 0.01154

> Box.test(resid(lm(detrain ~ detSOI, data = detsoi)), type = "Ljung-Box",
          + lag = 25)
```
Box-Ljung test

data: resid(lm(detrain ~ detSOI, data = detsoi))
X-squared = 37.92, df = 25, p-value = 0.04706

> Box.test(resid(lm(detrain ~ detSOI, data = detsoi)), type = "Ljung-Box",
+ lag = 30)

Box-Ljung test

data: resid(lm(detrain ~ detSOI, data = detsoi))
X-squared = 47.18, df = 30, p-value = 0.02391

The p-values are:

<table>
<thead>
<tr>
<th>n</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.0115</td>
</tr>
<tr>
<td>20</td>
<td>0.03325</td>
</tr>
<tr>
<td>25</td>
<td>0.0471</td>
</tr>
<tr>
<td>30</td>
<td>0.0239</td>
</tr>
</tbody>
</table>

The settings with \(n>15\) allow for more possibilities, with an accordingly reduced probability of detection, than do larger values of \(n\). The small p-value for \(n=30\) is perhaps surprising.