TWIN STRONGLY REGULAR GRAPHS: SOME QUESTIONS

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A simple graph Γ of order v is *strongly regular* [1] with parameters (v, k, λ, μ) if

- each vertex has degree k,
- each adjacent pair of vertices has λ common neighbours, and
- each nonadjacent pair of vertices has μ common neighbours.

Question 1. For which parameters (v, k, λ, μ) does there exist a regular graph G of order v and degree $2k$ that can be given a two-edge colouring (say red and blue) such that each of the red and blue subgraphs are strongly regular with parameters (v, k, λ, μ) and such that there exists an automorphism of G that swaps the two edge colours?

Example. The two-edge-coloured graphs $\Delta_m, M \geq 1$, defined in [2], form a sequence where each of the red and blue subgraphs of Δ_m are strongly regular with parameters

$$(\nu, k, \lambda = \mu) = (4^m, 2^{2m-1} - 2^{m-1}, 2^{2m-2} - 2^{m-1}).$$

For $m = 1, 2, 3$ it is relatively easy (e.g. using `iGraph`) to construct an automorphism of Δ_m that swaps the two colours. For $m > 3$ the problem is open.

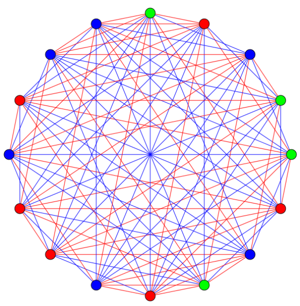

FIGURE 1. Δ_2

Figure 1 shows Δ_2, with each of the red and blue subgraphs being a $(16, 6, 2, 2)$ strongly regular graph. (Please ignore the vertex colouring.)

Question 2. As a special case of Question 1, restrict (ν, k, λ, μ) to $\nu = 4^m$, $k = 2^{2m-1} - 2^{m-1}$, $\lambda = \mu = 2^{2m-2} - 2^{m-1}$. In particular, for which m is there an isomorphism of Δ_m that swaps red and blue edges?

REFERENCES

[1] A. E. Brouwer, A. Cohen, and A. Neumaier. *Distance-Regular Graphs*. Ergebnisse der Mathematik und Ihrer Grenzgebiete, 3 Folge/A Series of Modern Surveys in Mathematics Series. Springer London, Limited, (2011).

[2] P. Leopardi, "Constructions for Hadamard matrices, Clifford algebras, and their relation to amicability / anti-amicability graphs", Australasian Journal of Combinatorics, Volume 58(2) (2014), pp. 214-248.

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