MAT 102 SPRING 2008
FEEDBACK ON HOMEWORK 4

This time I shall focus only on mistakes that are absolutely essential to avoid, say in your mid-term.

1. Some common mistakes.

- Always remember to simplify your answer when you work on homework problems: answers like $-\frac{1}{2}+\frac{1}{3}-(-2)-\frac{8}{3}$ are certainly not acceptable!
- **Q16 of §5.4.** It seems that many of you have difficulty with this question. There are two difficulties: First, in evaluating the indefinite integral: some of you seemed to be trying to apply some sort of 'quotient rule', which is terribly wrong: there are no quotient rule for integrals. In particular,

$$\int \frac{1+\cos 2t}{2} dt \neq \frac{\int (1+\cos 2t) dt}{\int 2dt}!$$

The correct way is to split the integral:

$$\begin{aligned} \int \frac{1+\cos 2t}{2} dt &= \int \frac{1}{2} dt + \frac{1}{2}\int \cos 2t dt \\ &= \frac{1}{2}t + \frac{1}{2}\left(\frac{\sin 2t}{2}\right) + C \\ &= \frac{t}{2} + \frac{\sin 2t}{4} + C. \end{aligned}$$

Next one needs to plug in the upper and lower limits of the integral:

$$\int_{-\pi/3}^{\pi/3} \frac{1+\cos 2t}{2} dt = \left(\frac{t}{2}+\frac{\sin 2t}{4}\right)\Bigg|_{-\pi/3}^{\pi/3}.$$

One just needs to be careful and remember that $\sin(2\pi/3) = \frac{\sqrt{3}}{2}$, $\sin(-2\pi/3) = -\frac{\sqrt{3}}{2}$. The answer is then $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$.
- **Q30 of §5.4.** When this question appears in this section, they suppose you to solve it 'the clever way': namely they expect you to somehow cleverly guess that

$$\int \frac{xdx}{\sqrt{1+x^2}} = \sqrt{1+x^2} + C.$$

However, now that you have learned substitution, there is a more standard way of doing it:

Let $u = 1 + x^2$. Then $du = 2xdx$ (or in other words, $xdx = \frac{1}{2}du$) so

$$\begin{aligned}\int \frac{xdx}{\sqrt{1+x^2}} &= \int \frac{1}{2}\frac{du}{\sqrt{u}} \\ &= \frac{1}{2}\int u^{-\frac{1}{2}}du \\ &= \frac{1}{2}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= \sqrt{u} + C \\ &= \sqrt{1+x^2} + C.\end{aligned}$$

Hence

$$\int_2^5 \frac{xdx}{\sqrt{1+x^2}} = \sqrt{1+5^2} - \sqrt{1+2^2} = \sqrt{26} - \sqrt{5}.$$

Make sure you understand how to do substitutions like this (or else, feel free to ask us) for questions about substitutions are bound to come up in the mid-term!

- **Q37 of §5.4.** Many of you still don't understand how to use the chain rule to differentiate those integrals that has a complicated upper/lower limits. Such questions again undoubtedly will come up in the mid-term... Here is an illustration how you could solve it. (Try applying them to the other problems!)

  To compute $\frac{d}{dx}\int_1^{\sin x} 3t^2 dt$, let $y = \int_1^{\sin x} 3t^2 dt$. We are going to compute $\frac{dy}{dx}$. Now let $u = \sin x$. Then $y = \int_1^u 3t^2 dt$, and by chain rule, we now have

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}.$$

  But

$$\frac{dy}{du} = 3u^2$$

  by the fundamental theorem of calculus, and $\frac{du}{dx} = \frac{d}{dx}\sin x = \cos x$. Hence

$$\frac{dy}{dx} = 3u^2 \cos x = 3(\sin x)^2 \cos x = 3\sin^2 x \cos x.$$

  Alternatively, if you like to write the chain rule using composition of functions, here is how you'd do it:

  Let $f(u) = \int_1^u 3t^2 dt$ and $g(x) = \sin x$. Then $\int_1^{\sin x} 3t^2 dt = f(g(x))$ so

$$\frac{d}{dx}\int_1^{\sin x} 3t^2 dt = f'(g(x))g'(x) = 3g(x)^2 g'(x) = 3\sin^2 x \cos x.$$

- **Q54 of §5.4.** Many of you didn't realize that the curve sometimes lies above the x-axis and sometimes below it. Therefore, to calculate the total area bounded by the curve between -2 and 2, one actually needs to figure out where the curve crosses the x-axis, i.e. one needs to solve $x^3 - 4x = 0$.

  Now $x^3 - 4x = 0$ if and only if $x(x^2 - 4) = 0$, i.e. if and only if $x(x+2)(x-2) = 0$. This is the same as saying that $x = 0, -2$ or 2. So we need to split the interval of integration $[-2, 2]$ into two halves (according to these zeroes), namely $[-2, 0]$ and $[0, 2]$. The area on the interval $[-2, 0]$ is

$$\left|\int_{-2}^0 (x^3 - 4x)dx\right| = |4| = 4$$

and the area on the interval $[0, 2]$ is

$$\left|\int_0^2 (x^3 - 4x)dx\right| = |-4| = 4.$$

Hence the total area is $4 + 4 = 8$.