MAT 102 SPRING 2008
HANDOUT 2 - SOME LITTLE TRICKS IN INTEGRATION

1. Little formula about d: useful for substitution and integration by parts

First some formula about trigonometric and exponential functions:

- $\cos x dx = d(\sin x)$
- $\sin x dx = -d(\cos x)$
- $\sec^2 x dx = d(\tan x)$
- $\csc^2 x dx = -d(\cot x)$
- $e^x dx = d(e^x)$
- $e^{-x} dx = -d(e^{-x})$

A little break: What are $e^{-2x}dx$, $\cos 2x dx$, etc, then?

Next, some formula about powers of x:

- $x dx = \frac{1}{2} d(x^2)$
- $x^m dx = \frac{1}{m+1} d(x^{m+1})$ if $m \neq -1$
- $\frac{dx}{\sqrt{x}} = 2d(\sqrt{x})$
- $\frac{dx}{x} = d(\ln x)$ (at least over the region $x > 0$).

Also, you can always stick in a little constant in all of these formula: like $\cos x dx$ can also be $d(\sin x - 1)$ or $\frac{1}{2} d(2 \sin x + 3)$ if you wish.

By the way, when you read these formula, you should really think about them: for instance, ask yourself: why are these formula true? How can you group them nicely so that you can remember them more easily? Are some of these repetitions of some others? Are there variations for these formula that would also be useful? When do you use each of these formula?

The thinking process involved would actually help you develop some problem solving skills that may be among the most important things that you will ever remember about this course.

2. trigonometric identities

List of useful trigonometric identities:

$$\begin{aligned}
\tan x &= \frac{\sin x}{\cos x} \\
\sin^2 x + \cos^2 x &= 1 \\
\tan^2 x &= \sec^2 x - 1 \\
\cot^2 x &= \csc^2 x - 1 \\
\sin 2x &= 2 \sin x \cos x \\
\cos^2 x &= \frac{1}{2}(1 + \cos 2x) \\
\sin^2 x &= \frac{1}{2}(1 - \cos 2x).
\end{aligned}$$

These are basically all the trigonometric identities that you will need for the course. The first two are essential; the third and the fourth are easy consequences of the first two and the definitions. I am afraid there is no easy way to remember the last three though, unless you learn something called 'the complex numbers'. We may touch upon that at a certain point, but not for now. By the way, these are just the identities without differentiation or integration; you will of course need to know the derivatives of these trigonometric functions.

Also, you will need to remember each of these formula forward and backward: for instance, you will need to know these formula well enough so that you can recognize $1+\tan^2 x$ as $\sec^2 x$ right away. Just try rewriting these formula in as many ways as you could by moving the terms around and you get used to this after a little while.