MAT 218 FALL 2008
REVIEW SESSION

GRE practice exam.

1. The minimal distance between any point on the sphere $(x-2)^2+(y-1)^2+(z-3)^2=1$ and any point on the sphere $(x+3)^2+(y-2)^2+(z-4)^2=4$ is
   (A) 0 (B) 4 (C) $\sqrt{27}$ (D) $2(\sqrt{2}+1)$ (E) $3(\sqrt{3}-1)$
2. Let C be the circle $x^2+y^2=1$ oriented counterclockwise in the xy-plane. What is the value of
   $$\int_C (2x-y)dx+(x+3y)dy?$$
   (A) 0 (B) 1 (C) $\frac{\pi}{2}$ (D) π (E)2π
3. Let F be a constant unit force that is parallel to the vector $(-1,0,1)$ in the xyz-space. What is the work done on a particle that moves along the path given by (t,t^2,t^3) between time $t=0$ and time $t=1$?
   (Mathematical rephrase: what is the path integral
   $$\int_C F\cdot dx$$
   where C is the path in the question?)
   (A) $-\frac{1}{4}$ (B) $-\frac{1}{4\sqrt{2}}$ (C) 0 (D) $\sqrt{2}$ (E) $3\sqrt{2}$
4. What is the minimum value of the expression $x+4z$ as a function defined on $\mathbb{R}^3$, subject to the condition $x^2+y^2+z^2\leq 2$?
   (A) 0 (B) -2 (C) $-\sqrt{34}$ (D) $-\sqrt{35}$ (E) $-5\sqrt{2}$

Miscellaneous problems.

1. Suppose that ϕ is a smooth function on $\mathbb{R}^2\setminus\{0\}$ and
   $$\phi_{xx}+\phi_{yy}+\phi_x=0$$
   there. Suppose also that in polar coordinates
   $$\begin{cases} r\phi_x(r,\theta)\to\cos\theta \\ r\phi_y(r,\theta)\to\sin\theta \end{cases}$$
   uniformly as $r\to 0$. Let C_R be the circle $x^2+y^2=R^2$, oriented counterclosewise in the xy-plane. Show that the line integral
   $$\int_{C_R} e^x(-\phi_y dx+\phi_x dy)$$
   is independent of R and evaluate it.
2. Let g be a smooth real-valued function on $(0,\infty)$ and let F be a vector field on $\mathbb{R}^3\setminus\{0\}$ defined by
   $$F(x)=g(|x|)x.$$
   Here $|x|$ denotes the Euclidean norm of the vector x. Let C be a smooth closed curve in $\mathbb{R}^3$ that does not pass through the origin. What can you say about the line integral
   $$\int_C F\cdot dx?$$

3. Compute the integral
$$\int\int_R \cos\left(\frac{x-y}{x+y}\right) dxdy$$
where R is the region of the xy-plane defined by $x \geq 0$, $y \geq 0$ and $x + y \leq 2$.
4. Let A be the region in the xyz-space defined by $x^2 + y^2 + z^2 \leq 4$ and $x^2 + y^2 \geq 1$. Let F be the vector field $(y, -x, z)$ on $\mathbb{R}^3$. Compute the surface integral
$$\int_{\partial A} F \cdot \nu d\sigma,$$
where ν is the outward unit normal to the surface ∂A. Relate this to the volume of A.

(Sources: Berkeley Problem Book, MIT opencourseware, and other online sources.)