MAT 330 SPRING 2009
PRACTICE EXERCISES FOR MIDTERM

Determine whether each of the following statements is true or false, and give a brief reason or a counterexample as is appropiate.

(1) If f is a continuous periodic function on $[0, 1]$, then its Fourier series converge pointwise at each point of $[0, 1]$.

(2) Let f be a periodic function that is twice differentiable and whose second derivative is continuous. Then its Fourier series converge absolutely.

(3) If the Fourier series of a periodic function converges absolutely, then it converges uniformly.

(4) If the Fourier series of a periodic function f converges at a point x_0 and if f is continuous at x_0, then the limit of the Fourier series at x_0 is equal to $f(x_0)$.

(5) Every continuous function on $[0, 1]$ is differentiable at at least one point in $(0, 1)$.

(6) If f is the periodic function on $[0, 1]$ defined by $f(x) = e^x$ for all $x \in [0, 1)$, then the Fourier series of f converges at every $x \in (0, 1)$.

(7)

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} e^{2\pi i n x}$$

is the Fourier series of a Riemann integrable periodic function on $[0, 1]$.

(8)

$$K_n(x) := \sum_{k=-\infty}^{\infty} \left(1 - \frac{1}{n}\right)^{|k|} e^{2\pi i k x}$$

is a good kernel on $[0, 1]$ as $n \to \infty$.

(9) The convolution of two continuous periodic function on $[0, 1]$ is continuous.

(10) If $\{\xi_n\}_{n=1}^{\infty}$ is equidistributed on $[0, 1]$, then $\{\xi_{2n}\}_{n=1}^{\infty}$ is also equidistributed on $[0, 1]$.

(11) There is a Schwartz function f on the real line that satisfies

$$\widehat{f}\left(\frac{x}{2}\right) = 2f(2x).$$

(12) The Fourier inversion formula

$$f(x) = \int_{-\infty}^{\infty} \widehat{f}(\xi) e^{2\pi i x \xi} d\xi$$

holds pointwise (with right hand side being a convergent integral at every x) if f is a smooth function on the real line with compact support.

(13) If f and g are functions on the real line and are both of moderate decrease then so is their product $f \cdot g$.

(14) If f and g are Schwartz functions on the real line, then

$$\int_{-\infty}^{\infty} \widehat{f}(\xi) \widehat{g}(\xi) e^{2\pi i x \xi} d\xi$$

is a convergent integral and is equal to $f * g(x)$ pointwise.

(15) Let $\mathbb{T}$ be the unit circle in the plane and $\mathbb{D}$ be the open unit disk it encloses. For any continuous function f on $\mathbb{T}$, there exists a function u on $\mathbb{D}$ such that u is twice continuously differentiable in $\mathbb{D}$, continuous on the closure of $\mathbb{D}$, and satisfies

$$\begin{cases} \Delta u = 0 & \text{in } \mathbb{D} \\ u = f & \text{on } \mathbb{T} \end{cases}.$$

(16) If f is a Schwartz function on the real line and $\widehat{f}(\xi) = 0$ for all $\xi \in [-1, 1]$, then

$$\int_{-\infty}^{\infty} |f(x)|^2 dx \leq C \int_{-\infty}^{\infty} |f'(x)|^2 dx$$

for some absolute constant C (that does not depend on f).