**Math 244 Fall 2012**

**Homework 1**

You should solve these problems without using a calculator.

1. Find the derivatives of the following functions of $t$ (wherever they are defined).

$$\cos(2t), \qquad te^{-t^2}, \qquad \sec^2(3t), \qquad \frac{t}{1+9t^2}, \qquad \frac{1}{\sqrt{1-4t^2}}$$

2. Find the anti-derivatives of the functions in Question 1.

3. Find all functions $y(t)$ satisfying

$$\frac{dy}{dt} = \cos^3(2t) + t\sin t.$$

4. Find all functions $y(t)$, defined for $t > 0$, such that

$$\frac{dy}{dt} = t\log t, \quad \text{and} \quad y(1) = 3.$$

5. (a) Sketch a graph of $y(t) = \sin(2t)$ for $0 \leq t \leq 2\pi$.
   (b) Where is $y(t)$ increasing within the above range of $t$?
   (c) Repeat the above if now $y(t) = t\sin(2t)$ instead.

6. This is a question about complex numbers. We write $i$ so that $i^2 = -1$, and use $|\cdot|$ to denote the modulus of a complex number. We *define*

$$e^{x+iy} = e^x(\cos y + i\sin y)$$

if $x$, $y$ are real numbers.

   (a) Show that $(a+bi)(a-bi) = a^2 + b^2$.
   (b) Compute $|-3+4i|$ and $|e^{-100i}|$.
   (c) Find the real and imaginary parts of $e^{x-iy}$, if $x$, $y$ are real numbers.
   (d) Define a function $y(t) = e^{(2-3i)t}$ for real values of $t$. Compute $y'(t)$ and evaluate that when $t = 0$.
   (e) Suppose $a$, $b$ are real numbers, and define two functions

$$y_1(t) = e^{(a+bi)t} + e^{(a-bi)t}, \quad \text{and} \quad y_2(t) = -i(e^{(a+bi)t} - e^{(a-bi)t})$$

for real values of $t$. Show that they are both real-valued functions of $t$, and that

$$y_1(t) = 2e^{at}\cos(bt), \quad \text{and} \quad y_2(t) = 2e^{at}\sin(bt).$$

7. Solve the following quadratic equations:

$$r^2 - 4r + 3 = 0, \qquad r^2 - 4r + 4 = 0, \qquad r^2 - 4r + 5 = 0.$$

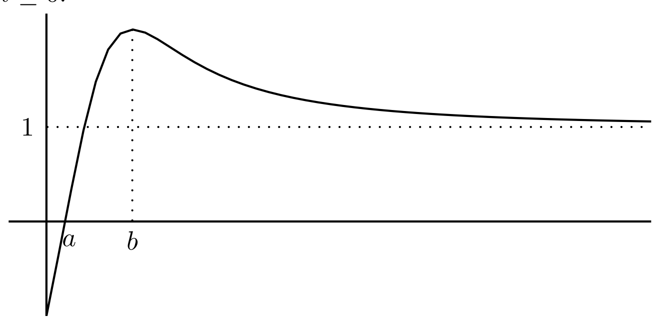
8. Find the Jacobian matrix of the following map at the point $(x, y)$:

$$F(x, y) = (-3\sin x + y, 4x + 3\cos y - 3).$$

9. Find the Jacobian matrix of the following map at the point $(1, 1)$:

$$F(x, y) = (2x - x^2 - xy, 3y - y^2 - 2xy).$$

10. In the following figure, the horizontal axis is the $t$-axis, and the vertical axis is the $y$-axis. The solid curve is the graph of $y = f(t)$ for some function $f(t)$, defined for $t \geq 0$.




Based on what is suggested in the graph:

(a) Decide the range of $t$ where $f(t)$ is positive.

(b) Decide the range of $t$ where $f'(t)$ is positive.

(c) Predict whether

$$\lim_{t\to+\infty} f(t) \quad \text{and} \lim_{t\to+\infty} f'(t)$$

exist, and what their values are if they exist.

11. Compute the following limits:

$$\lim_{t\to\infty} \frac{1 + 5t - 4t^2}{1 + 3t^2}, \qquad \lim_{t\to\infty} \frac{\sin t}{t}, \qquad \lim_{t\to\infty} \frac{\log(1 + t^2)}{t(t + 1)}.$$

12. For each of the following statements, decide whether it is true or false. Give a counter-example if the statement is false.

(a) If $f(t) \geq 0$ for all $a < t < b$, then $f(t)$ is increasing on the interval $(a, b)$.

(b) If $f'(t) \geq 0$ for all $a < t < b$, then $f(t)$ is increasing on the interval $(a, b)$.

(c) If $f''(t) \geq 0$ for all $a < t < b$, then $f(t)$ is increasing on the interval $(a, b)$.