Math 350 Fall 2011
Homework 9 Additional Problems

1. Suppose A is a 3×3 matrix with entries in $\mathbb{R}$, and that
$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$$
are eigenvectors of A. Is A diagonalizable? Explain.
2. Suppose P_2 is the vector space of all real polynomials of degree ≤ 2 over $\mathbb{R}$, and $T\colon P_2 \to P_2$ is a linear map such that $1 + x$, $x + x^2$ and $1 + 4x + 4x^2$ are eigenvectors of T. Is T diagonalizable? Explain.
3. Suppose
$$v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
and A is a 2×2 matrix with entries in $\mathbb{R}$ such that
$$Av_1 = 3v_1$$
and
$$Av_2 = -6v_2.$$
Find the matrix A.
4. Suppose P_2 is the vector space of all real polynomials of degree ≤ 2 over $\mathbb{R}$, and $T\colon P_2 \to P_2$ is a linear map such that
$$T(1) = 1, \quad T(x-1) = 7(x-1) \quad \text{and} \quad T((x-1)^2) = 7(x-1)^2.$$
Show that
$$T(p(x)) = p(x) + 3(x-1)^2 p''(x) \quad \text{for all } p(x) \in P_2.$$