Math 403 Spring 2011
Homework 2 Additional problems

1. Let D be the open unit disc $\{z \colon |z - i| < 1\}$. For each of the following choices of $f(z)$, compute $\dfrac{\partial f}{\partial z}$ and $\dfrac{\partial f}{\partial \overline{z}}$ on D. Which of them are analytic on D? For those, compute $f'(z)$ on D, and find an analytic function $F(z)$ on D such that $F'(z) = f(z)$ on D. You should explain why your F is analytic on D.

(a) $f(z) = z + \dfrac{2i}{z^2}$

(b) $f(z) = (z^2 - 7i)|z|^2$

(c) $f(z) = (i\overline{z}^3 - 4\overline{z} + 1)^5$

(d) $f(z) = \dfrac{(z^2 + i)^2}{z^2}$

(e) $f(z) = |\overline{z}^3 - iz^2|^2$

(f) $f(z) = \dfrac{(iz + 1)(iz + 2)|z|^4}{\overline{z}^2}$