Harmonic Analysis Workshop Abstracts

August 27, 2024

Neal Bez
On the ubiquity of geometric Brascamp–Lieb data

The Brascamp–Lieb inequality unifies several important inequalities such as Hölder's inequality, Young's convolution inequality and the Loomis–Whitney inequality. A particularly important special case is the geometric Brascamp–Lieb inequality. This inequality goes back to pioneering work of Keith Ball in the 1980s in convex geometry, and it turns out to play a fundamental role in the general theory of the Brascamp–Lieb inequality. For instance, it was shown by Bennett, Carbery, Christ and Tao that a Brascamp–Lieb inequality which possesses maximizers is equivalent to a geometric Brascamp–Lieb inequality. Relying heavily on work of Garg, Gurvits, Oliveira and Wigderson, here we present another sense in which the class of geometric Brascamp–Lieb data may be considered large. This addresses a question of Bennett and Tao in their recent work on the adjoint Brascamp–Lieb inequality. Joint work with Anthony Gauvan and Hiroshi Tsuji.

Michael Christ
Implicitly Oscillatory Multilinear Integrals and Associated Sublevel Set Inequalities

Multilinear oscillatory integrals arise in various contexts in harmonic analysis, in partial differential equations, in ergodic theory, and in additive combinatorics. We discuss the majorization of integrals $\int \prod_j (f_j \circ \varphi_j)$ of finite products by negative order Sobolev norms of the factors, where integration is over a ball in Euclidean space and φ_j are smooth mappings to a space of strictly lower dimension. Several applications will be indicated. The talk focuses on the quadrilinear case, after work on the trilinear case of Bourgain

(1988), of Joly, Metivier, and Rauch (1995), the speaker (2019), and others. Sublevel set inequalities, which quantify the nonsolvability of certain systems of linear equations, are a central element of the analysis.

Polona Durcik
Quantitative norm convergence of triple ergodic averages for commuting transformations

We discuss a quantitative result on norm convergence of triple ergodic averages with respect to three general commuting transformations. For these averages we prove an r-variation estimate, $r > 4$, in the norm. We approach the problem via real harmonic analysis, using the recently developed techniques for bounding singular Brascamp-Lieb forms. It remains an open problem whether such norm-variation estimates hold for all $r \geq 2$ as in the cases of one or two commuting transformations, or whether such estimates hold for any $r < \infty$ for more than three commuting transformations. This is joint work with Lenka Slavikova and Christoph Thiele.

Alexander Fish
On the volumes of simplices in sets of positive density in $\mathbb{Z}^d$

The following is a classical problem in geometric Ramsey theory: What are the n-point configurations that appear in every subset of positive density of $\mathbb{Z}^d$. In this talk we will discuss the dynamical approach to this general problem. We will also present the main ideas behind our recent result that for any set E of positive density in $\mathbb{Z}^d$, the set of volumes of all simplices generated by the points in E contains an infinite arithmetic progression. The talk is based on a joint work with Michael Bjorklund (Chalmers).

Andrew Hassell
Rough waves and Hardy spaces associated to Fourier integral operators

I will talk about work with Portal, Rozendaal and Yung on function spaces adapted to Fourier integral operators, and their utility in solving rough wave equations with data in L^p spaces.

Bryce Kerr
Moments of the Riemann zeta function and decoupling inequalities

A result of Heath-Brown gives an upper bound for the 12-th moment of the Riemann zeta function on the critical line which is derived from suitable large value estimates. In this talk I'll explain how Heath-Brown's argument can be refined so that decoupling inequalities of the type considered by Bourgain (J. d'Analyse Mathematique 2017) can be used as input after an application of the 'double large sieve'. In particular, this implies some new large value estimates for the Riemann zeta function on the critical line. I'll also discuss some of the fundamental barriers in this problem that a direct application of decoupling fails to bypass.

Changkeun Oh
Discrete restriction estimates for manifolds avoiding a line.

l^2 discrete restriction estimates are inequalities estimating exponential sums. Bourgain and Demeter proved the l^2 decoupling inequalities for the paraboloid and hyperbolic paraboloid, which easily imply the optimal l^2 discrete restriction estimates for the paraboloid and hyperbolic paraboloid. By the geometric fact that the hyperbolic paraboloid contains a line, the optimal l^2 discrete restriction estimate for the hyperbolic paraboloid is worse than that for the paraboloid. Based on this observation, one may expect that if a hypersurface does not contain a line, then it might be possible to obtain the l^2 discrete restriction estimate for the hypersurface as good as that for the paraboloid. In the joint work with Guth and Maldague, we confirmed this by proving the l^2 discrete restriction estimates for the manifolds avoiding a line.

Pierre Portal
Maximal regularity of parabolic problems

Regularity questions for linear parabolic PDE and SPDE boil down to harmonic analysis questions about the boundedness of singular integral operators acting on function spaces over space-time. When coefficients are sufficiently smooth, these operators can be studied using Calderon-Zygmund (CZ) theory. At the level of irregularity required by current PDE theory (and even more so for SPDE), however, the operators are outside of the CZ class. Nevertheless, they can be studied by extending the principles of CZ theory. In this talk, I will explain how such an extension works, and present recent breakthroughs (joint work with Pascal Auscher) that allow us to prove regularity for both deterministic and stochastic problems with L^∞ coefficients (in time, space, and the probability variable) with L^p data. When $p = 2$, such results have been known since the 1950s. For $p \neq 2$, however, all previous results required some regularity, either in space, or in time. I will briefly touch on the PDE and SPDE challenges of this problem, but will focus on the harmonic analysis, and especially on the principles that can be used elsewhere.

Alexandria Rose
The Hyperbolic Fractal Uncertainty Principle

The fractal uncertainty principle qualitatively states that a function and its Fourier transform cannot both concentrate on a (regular) fractal set. More precisely, if for small $h > 0$, we let $\mathcal{F}_h$ denote the semiclassical Fourier transform, $\mathcal{F}_h f(\xi) = \frac{1}{\sqrt{2\pi h}} \int_{\mathbb{R}} e^{-ix\cdot\xi/h} f(x)\mathrm{d}fx$, we say that the sets X and Y satisfy a fractal uncertainty principle with exponent β if

$$\|\mathbb{1}_{X_h} \mathcal{F}_h \mathbb{1}_{Y_h}\|_{L^2\to L^2} \lesssim h^\beta$$

for some nontrivial exponent β, where here X_h and Y_h denote the h-neighborhoods of X and Y. Fractal uncertainty principles were first used to answer questions in quantum chaos about the behavior of chaotic billiards on negatively curved manifolds and to obtain estimates for the eigenfunctions of the Laplacian on such manifolds. Expressing these manifolds as a hyperbolic quotient, one sees that the group structure of the quotient generates a fractal set which captures the limiting behavior of chaotic trapped particles. In this talk, we will explore a more generalized version of the fractal uncertainty principle in which the Fourier transform is replaced by a Fourier integral operator with a more general phase. An important model case of special interest is the hyperbolic phase $\Phi(x, y) = \log |x - y|$. We prove new bounds for the hyperbolic fractal uncertainty principle that holds for fractal sets even in cases in which there is no usual (linear) fractal uncertainty principle, demonstrating the fundamental difference between the hyperbolic and the linear case. Joint work with Francisco Romero Acosta.

Robert Schippa
Quantified decoupling estimates and applications

We start with discussing square function estimates for curves of finite type and applications to Strichartz estimates for (quasi-)periodic functions. Next, we shall see how arguments from decoupling theory combined with semi-classical Strichartz estimates allow for the proof of trilinear Strichartz estimates on frequency-dependent times. Related arguments allow us to improve Bourgain's L^2-well-posedness result for the periodic KP-II equation proved in 1993. The last part of the talk is joint work with S. Herr and N. Tzvetkov.

Igor Shparlinski
Maximal Operators and Restriction Bounds for Weyl Sums

We describe several recent results on so called maximal operators on Weyl sums

$$S(u; N) = \sum_{1 \le n \le N} \exp(2\pi i(u_1 n + \ldots + u_d n^d)),$$

where $u = (u_1, ..., u_d) \in [0,1)^d$. Namely, given a partition $I \cup J \subseteq \{1, \ldots, , d\}$, we define the map $[0,1)^k \to [0, N]$, where $k = \#I$, as

$$(u_i)_{i \in I} \mapsto \sup_{u_j,\, j \in J} |S(u; N)|$$

which corresponds to the maximal operator on the Weyl sums associated with the components u_j, $j \in J$, of u. We are interested in understanding this map for almost all $(u_i)_{i \in I}$ and also in the various norms of these operators.

Questions like these have several surprising applications, including outside of number theory, and are also related to restriction theorems for Weyl sums.

Adam Sikora
Bochner-Riesz profile of harmonic oscillator, anharmonic oscillator and Laguerre expansions

We start with discussion of spectral multipliers and Bochner-Riesz means corresponding to the Schrödinger operator with anharmonic potential $\mathcal{L} = -\frac{d^2}{dx^2} + |x|$. We show that the Bochner-Riesz profile of the operator $\mathcal{L}$ completely coincides with such profile of the harmonic oscillator $\mathcal{H} = -\frac{d^2}{dx^2} + x^2$. Then we extend our discussion to include order α Laguerre expansion corresponding to the operator $\mathcal{H}_\alpha = -\frac{d^2}{dx^2} - (2\alpha + 1)\frac{d}{dx} + x^2$, which can be interpreted as radial part of multidimensional harmonic oscillators.

Based on joint a work with Peng Chen, and Waldemar Hebisch and a current project with Himani Sharma and Sundaram Thangavelu.

Aleksander Simonič
The Guinand-Weil formula and conditional estimates for L-functions

I am going to give an overview on conditional (G(RH)) estimates for moduli of the logarithmic derivative and the logarithm of zeta and other L-functions in the critical strip. In modern estimates, sums over the non-trivial zeros are bounded via the Guinand-Weil exact formula, and this part of the technique is related to harmonic analysis.

Melissa Tacy
Simultaneous saturation and multilinear restriction estimates

A classical problem of harmonic analysis is to prove L^p estimates for oscillatory integral operators. Typically L^2 estimates are relatively easy to obtain due to orthogonality allowing us to treat oscillations in different directions as essentially independent. Conversely L^∞ results are often easy, or at least tractable, because they represent the worst case of interactions between oscillations in different directions. Obtaining estimates for the values of p in-between is frequently a very hard problem. In this talk I will present a new technique which I have been developing, simultaneous saturation. Here one considers the worst case L^∞ growth and uses geometric averages to prove bounds that limit the number of points at which one can simultaneously saturate the L^∞ bounds. I will apply these techniques to demonstrate a re-proof of the Multilinear Restriction Theorem which can be extended to deal with parameter dependent multilinear models.

Christoph Thiele
Quantum signal processing and nonlinear Fourier analysis

We will describe how a recently studied algorithm to represent a signal by a quantum computing device is related to the rather classical subject of nonlinear Fourier analysis. Transference of ideas between the subjects has resulted in progress on both. This is joint work with Michel Alexis and Gevorg Mnatsakanyan as well as in part with Lin Lin and Jiasu Wang.

Timothy Trudgian
Infernal Kernels? Eternal Journals!

Thirty years ago Hugh Montgomery presented ten lectures on the interface between number theory and harmonic analysis. I will discuss some choices of kernels and special polynomials, which are mentioned in these lectures, and which continue to provide rich research problems to this day.

Nina Zubrilina
Murmurations of modular forms

In a recent machine learning-based study, He, Lee, Oliver, and Pozdnyakov observed an unexpected oscillating pattern in the average value of the P-th Frobenius trace of elliptic curves of prescribed rank and conductor in an interval range. Sutherland later discovered that this bias extends to Dirichlet coefficients of other classes of L-functions when split by root number. In my talk, I will prove this bias for a family of holomorphic modular forms and for a family of Maass forms.