Yamabe problems

We studied the k-Yamabe problem, which can be reduced to the existence of solutions to the conformal k-Hessian equation on manifold. The classical Yamabe problem ($k = 1$) was resolved by Yamabe, Trudinger, Aubin, and finally by Schoen. Many people contributed to the k-Yamabe problem for $k \geq 2$.

By some techniques from our previous research on the k-Hessian equation, we resolved the k-Yamabe problem for $k = 2$ by a variational approach [yp1] (also by Chang-Gursky-Yang when $n = 4$ and by Y. Ge-G. Wang when $n \geq 8$, where n is the dimension of the manifold). The case $k > n/2$ was solved by Gursky-Viaclovsky, but we proved a stronger result, namely the set of all normalized admissible metrics is compact [yp2]. In a recent paper [yp3], we are able to solve the problem for $k = \frac{n}{2}$. The cases $2 < k < \frac{n}{2}$ is still open except when the manifold is locally conformally flat, which was solved by P.Guan-G.Wang, and A.Li-Y.Li, and later also in [yp1], by different methods.

In all the above works, the initial metric is assumed to be k-admissible, so that the equation is elliptic. When $k \geq 3$, Branson and Gover proved that the k-Yamabe problem is variational only when the manifold is locally conformally flat. The interior gradient estimate of P.Guan and G.Wang (also different proofs by S. Chen, Y. Li, and myself [yp4]) and the Liouville theorem by A. Li and Y. Li played an important role in our papers.

References

[yp1] W.M. Sheng, N.S. Trudinger, X.-J. Wang, Yamabe problem for higher order curvatures, J. Diff. Geom. 77(2007), 515-553.

[yp2] N.S. Trudinger, X.-J. Wang, Harnack inequalities and singularities for admissible metrics in Yamabe problem, Calc. Var. PDE, 35(2009), 317-338.

[yp3] N.S. Trudinger, X.-J. Wang, The intermediate case of the Yamabe problem for higher order curvatures, in preparation.

[yp4] X.-J. Wang, A priori estimates and existence for a class of fully nonlinear equations in conformal geometry, Chinese Ann. Math., 27(2006), 169-178.