Prescribing curvature equations

1. Gauss curvature.

In [pc4] we resolved the conjecture of Spruck at the 1994 International Congress of Mathematicians, that is the existence of locally convex hypersurface of constant Gauss curvature. It is an analogue of the Plateau problem for the Gauss curvature. A similar proof was simultaneously found by B. Guan and J. Spruck.

In [pc3] we re-proved the existence of smooth solutions to the Minkowski problem by a Gauss curvature flow approach. The Minkowski problem was previously solved by Pogorelov and Nirenberg in dim $n = 2$, and by Pogorelov and Cheng-Yau for all n. An extended Minkowski problem, proposed by Lutwak, was treated in [pc7].

We also proved [pc1] the existence of a convex hypersurface in $\mathbb{R}^{n+1}$ whose Gauss curvature is equal to a given positive function $f \in C^2(\mathbb{R}^{n+1})$, provided f converges to a positive constant at infinity. Previous works assume local conditions of f.

2. k-curvature.

For a hypersurface $M \subset \mathbb{R}^{n+1}$, the k-curvature is the k^{th} elementary symmetric polynomial of the principal curvatures of M. It is respectively the mean curvature, scalar curvature, and the Gauss curvature when $k = 1, 2$, and n.

In [pc5] we proved the interior second derivative estimate of Pogorelov type for the k-curvature equation (higher regularity follows from Evans and Krylov's regularity theory). The interior gradient estimate was proved by Korevaar but a simpler proof was given in [pc2]. The global regularity for the Dirichlet problem was proved by Caffarelli, Nirenberg, and Spruck for vanishing boundary condition, and by Ivochkina for general boundary condition. For the curvature quotient equations, the global regularity was obtained by Ivochkina and Trudinger. A monotonicity formula for the k-curvature equation was proved by Urbas. In [pc6] we also contributed to the existence of closed convex hypersurface of prescribed k-curvature.

References

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